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ABSTRACT

This is part of a series of 42 Calculus Based Physics (CBP) modules totaling about 1,000 pages. The modules include study guides, practice tests, and mastery tests for a full-year individualized course in calculus-based physics based on the Personalized System of Instruction (PSI). The units are not intended to be used without outside materials; references to specific sections in four elementary physics textbooks appear in the modules. Specific modules included in this document are: Module 18--Sound; Module 19--Temperature, Heat, and Thermodynamics: First Law; and Module 20--Kinetic Theory of Gases. (CP)

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STUDY MODULES FOR CALCULUS-BASED GENERAL PHYSICS*

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SE 020 454

Comments

These modules were prepared by fifteen college physics professors for use in self-paced, mastery-oriented, student-tutored, calculus-based general physics courses. This style of teaching offers students a personalized system of instruction (PSI), in which they increase their knowledge of physics and experience a positive learning environment. We hope our efforts in preparing these modules will enable you to try and enjoy teaching physics using PSI.

Robert G. Fuller
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These modules were prepared by the module authors at a College Faculty Workshop held at the University of Colorado - Boulder, from June 23 to July 11, 1975.

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COMMENT TO USERS

In the upper right-hand corner of each Mastery Test you will find the "pass" and "recycle" terms and a row of numbers "1 2 3 ..." to facilitate the grading of the tests. We intend that you indicate the weakness of a student who is asked to recycle on the test by putting a circle around the number of the learning objective that the student did not satisfy. This procedure will enable you easily to identify the learning objectives that are causing your students difficulty.

COMMENT TO USERS

It is conventional practice to provide several review modules per semester or quarter, as confidence builders, learning opportunities, and to consolidate what has been learned. You the instructor should write these modules yourself, in terms of the particular weaknesses and needs of your students. Thus, we have not supplied review modules as such with the CBP Modules. However, fifteen sample review tests were written during the Workshop and are available for your use as guides. Please send $1.00 to CBP Modules, Behlen Lab of Physics, University of Nebraska - Lincoln, Nebraska 68588.

FINIS

This printing has completed the initial CBP project. We hope that you are finding the materials helpful in your teaching. Revision of the modules is being planned for the Summer of 1976. We therefore solicit your comments, suggestions, and/or corrections for the revised edition. Please write or call

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STUDY GUIDE

SOUND

INTRODUCTION

We, much more than our ancestors, are constantly being bombarded by sound. We hear, and are more or less aware of, music, air hammers, television sets, jet planes, conversation, engines, sirens, etc., throughout the day. A sophisticated audio industry tries to improve the quality of musical sound. At the other end of the quality scale, noise pollution is a serious concern, which probably affects our lives more than we realize.

In order to deal with sound, we should have some idea of what it is. What factors determine whether a sound is pleasing or grating? How is sound transmitted? How much power do our ear drums actually receive when we hear a bird, or a rock band? What determines whether our ears detect a sound; what is the meaning of sound beyond the range audible to a human? These are just a sample of a multitude of physical and physiological questions that one might ask about sound. Not all of these questions will be answered in this module, but you will learn enough to begin finding answers to several of them.

PREREQUISITES

Before you begin this module, you should be able to:	Location of Prerequisite Content
*Describe the motion of a transverse wave on a string (needed for Objective 1 of this module)	Traveling Waves Module
*Find the partial derivative of a function of two variables (needed for Objective 2 of this module)	Partial Derivatives Review
*Apply Newton's second law to problems in one dimension (needed for Objective 2 of this module)	Newton's Laws Module
*Write the meaning and give an example of the following terms: frequency, angular frequency, amplitude, and phase as they apply to simple harmonic motion (needed for Objectives 1 to 4 of this module)	Simple Harmonic Motion Module

LEARNING OBJECTIVES

After you have mastered the content of this module, you will be able to:

1. Definitions and relations - Explain what sound is, describe the physical significance of displacement amplitude, pressure amplitude, and intensity, and describe the relationship between pressure and particle displacement.

2. Mathematical description - Given an expression for pressure or displacement, determine whether it represents a possible traveling sound wave, and if so, determine the characteristics of the wave.

3. Superposition - Apply the principle of superposition to determine the characteristics of the sound at a point produced by two sources.

4. Resonance - Calculate the conditions for resonance in a given open or closed air column.

5. Doppler shift - Calculate the effects on the frequency of a sound wave produced by motion of the source and/or the receiver with respect to the medium.

GENERAL COMMENTS

We are fortunate that we don't need to learn entirely new physics, and entirely new methods of analysis, for each of the many different types of waves. Although there are important differences to be found among electromagnetic, elastic, hydrodynamic, and gravitational waves, it is the striking similarities among them that make the subject of wave motion more comprehensible.

You have already studied transverse waves on a string in the module Traveling Waves. Your study of other kinds of wave phenomena will be simplified if you concentrate on the similarities between the waves you study and waves on a string. We mention here some of the important similarities.

1. Wave Equation

All waves are mathematically similar. The mathematical expression for the displaced quantity is a solution of a particular partial-differential equation, the wave equation. The solutions of that equation describe the wave at any position and at any instant of time. In one dimension, the solutions of the wave equation are functions of both variables, x and t. In simple traveling waves, however, x and t always appear in a particular combination:

$$f(x, t) = f(x \mp vt),$$

where v is the wave speed. The minus sign is associated with waves moving in the positive x direction, and the plus sign is associated with waves moving in

the negative x direction. To understand this a little better, consider a wave in the shape of a pulse, as shown in Figure 1. The first graph (a) shows the pressure at a particular instant, t = 0. We could describe that wave form by some function, p = p(x).

The wave moves essentially without distortion to the right, at the wave velocity v. The second graph (b) shows the pulse at the later time, $t = t_1$, when the pulse has moved a distance vt_1. Since the shape of the wave is unchanged, the functional form describing the wave must be the same, but translated to the right a distance vt_1. We can write $p = p(x')$, where the origin of the x' reference frame is translated a distance vt_1 to the right,

$$x' = x - vt_1.$$

Therefore, for any time, the pressure is described by a functior $p = p(x - vt)$. If the pulse had been moving to the left, we would have changed the sign of vt in the argument.

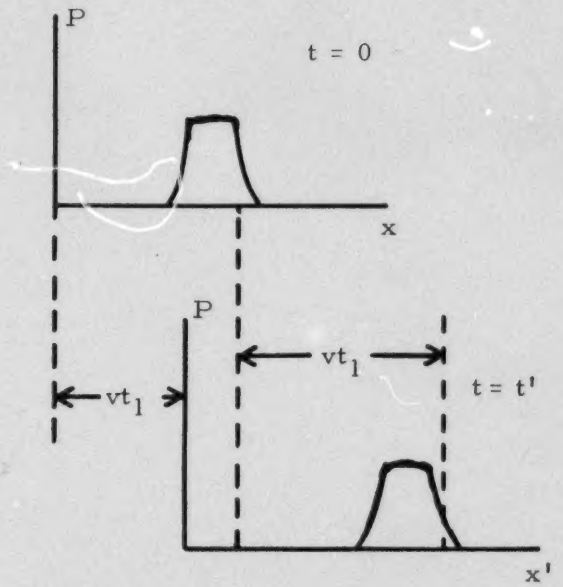


Figure 1

2. Principle of Superposition

If $f_1(x - vt)$ and $f_2(x - vt)$ are two different solutions of the wave equation, corresponding to different waves, the sum of these two functions is also a solution, if the amplitudes are not too large. Physically, this means that complicated wave disturbances can be broken down into a superposition of simpler disturbances. This fact is called the principle of superposition. You will be studying a few of the consequences of that principle.

One important application of the principle of superposition is the addition of two waves of slightly different frequencies. Let us assume that two sources emit sinusoidal waves with angular frequencies ω_A and ω_B. If the sound arriving at a particular point from each source has the same amplitude, the individual disturbances can be written as

$$y_A = y_0 \sin \omega_A t \quad \text{and} \quad y_B = y_0 \sin \omega_B t.$$

The sum of these two waves is then

$$y = y_0(\sin \omega_A t + \sin \omega_B t).$$

If we now apply the trigonometric identity

$$\sin a + \sin b = 2 \cos[(a - b)/2] \sin[(a + b)/2],$$

we get

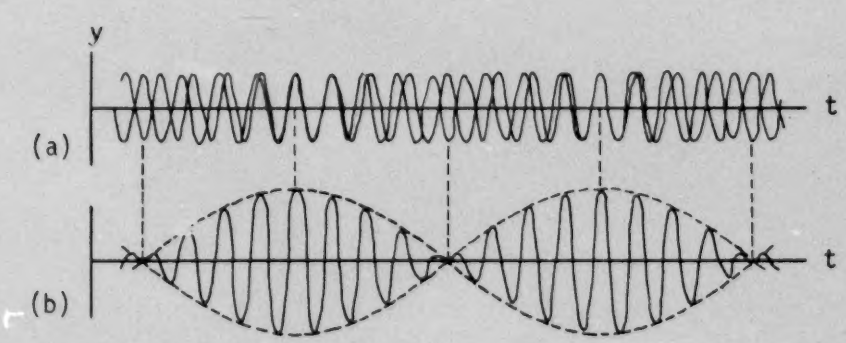
$$y = 2y_0 \sin[(\omega_A + \omega_B)t/2] \cos[(\omega_A - \omega_B)t/2].$$

If ω_A and ω_B are nearly the same, the disturbance is the product of a rapidly varying term with frequency $(\omega_A + \omega_B)/2$, and a slowly varying term with frequency $(\omega_A - \omega_B)/2$. Figure 2 shows two waves with slightly different frequencies in (a), and in (b) the sum of the two waves. The slowly varying term acts as an envelope that determines the amplitude of the rapidly oscillating part. The maximum amplitude occurs when

$$\cos[(\omega_A - \omega_B)t/2] = \pm 1.$$

There are, therefore, two <u>beats</u> in each cycle of the amplitude term, thus the beat frequency is $\omega_A - \omega_B$.

Figure 2



The beat phenomenon is true for all kinds of waves (electromagnetic, sound, water, etc.) at all but the highest intensities. In sound, musicians will frequently tune to each other by playing the same note and tuning out the beats. The human ear can detect beats between two tones up to a frequency of about 7.0 Hz. Highly skilled piano tuners, by tuning out beats, can tune a piano more accurately than all but the most sophisticated electronic tuners.

3. Pressure

In a sound wave, the dependent variable may be chosen to be either the displacement of a particle in the medium, or the deviation of the pressure from its equilibrium value. In either case the variable is a function of both x and t, and satisfies the wave equation. The solution to the wave equation can always be expressed in the form of a function of $(x + vt)$, depending on whether the wave is moving in the direction of decreasing or increasing x, or as a sum of functions of $(x + vt)$. The physical process is that a change in pressure produces a change in the volume of an element of mass of the medium. Since the change in volume is related to the particle displacement, the pressure and displacement are related. Either can be chosen as the dependent variable in describing a sound wave. Usually the pressure is more convenient to work with.

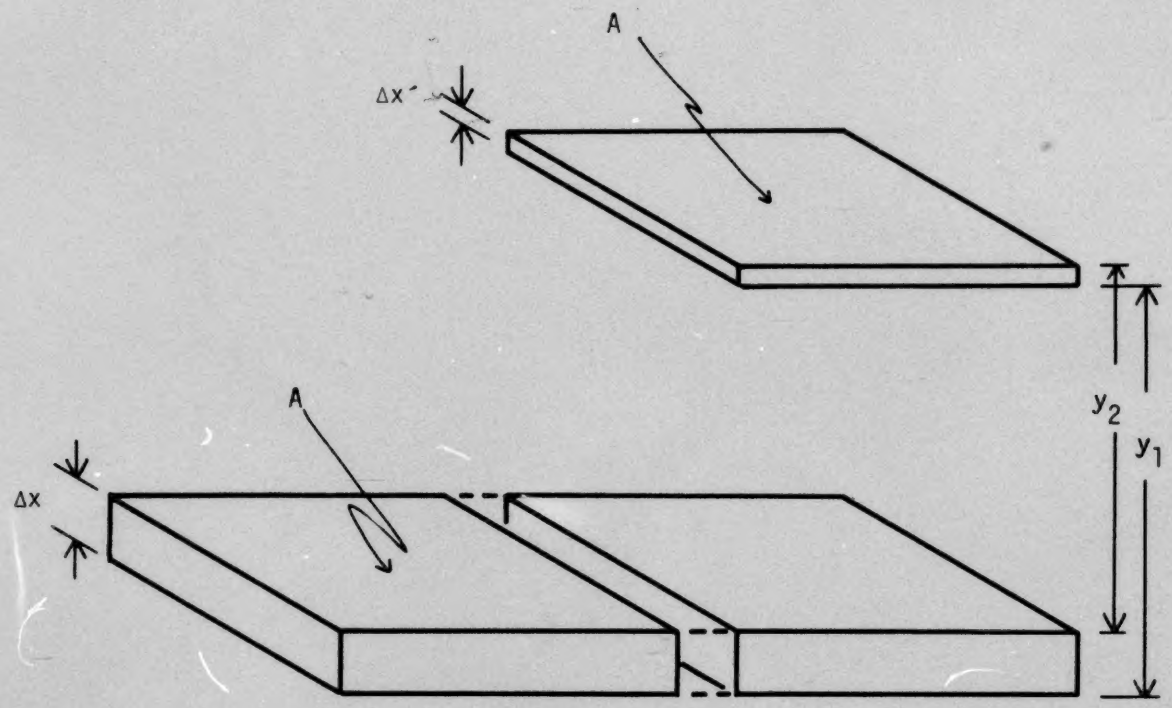


Figure 3

Figure 3 shows a thin slab of medium, containing mass M of the material at equilibrium with pressure p_0. Since the element of the medium is at equilibrium, there is no net force on the element and no acceleration. If a plane sound wave now passes through the region of space that includes the element, the volume containing M oscillates about its equilibrium position. Since the pressure changes, the volume of the element will also change, dependent on position and time.

Suppose that at a particular instant of time the element has been displaced to the position shown in Figure 3. Notice that the element has also been compressed. A particle at the left-hand end of the element has been displaced at distance y_1, and a particle at the right-hand end has moven y_2. The pressure at the left side of the element is $p + p_0$. If we call the displacement of the center of mass from its equilibrium position y, then Newton's second law for this element is

$$\Sigma F_i = Ma = (\rho A\ \Delta x')(d^2y/dt^2), \tag{1}$$

where ρ is the density of the medium. Since all forces are in one direction, we can omit the vector notation. The forces on the element in its displaced position are

$$\Sigma F_i = (p_0 + p)A - (p_0 + p + \Delta p)A = -\Delta pA,$$

and Eq. (1) becomes

$$-\Delta pA = \rho A\ \Delta x'(\partial^2 y/\partial t^2), \qquad \Delta p = -\rho\ \Delta x'(\partial^2 y/\partial t^2). \tag{2}$$

To get an expression in terms of one variable instead of both pressure and displacement, we employ the bulk modulus:

$$B = -\frac{\Delta p}{(\Delta V'/V)},$$

where $V = A\ \Delta x$ and $\Delta V = A(\Delta x' - \Delta x) = -A(y_2 - y_1) = -A\ \Delta y$. Thus

$$\Delta V/V = -\Delta y/\Delta x \quad \text{and} \quad \Delta p = -B(\Delta y/\Delta x).$$

Substituting for Δp in Eq. (2) we find

$$\Delta p = -B\frac{\Delta y}{\Delta x} = -\rho\ \Delta x'\frac{\partial^2 y}{\partial t^2} \xrightarrow{\Delta x,\ \Delta x' \to 0} \frac{\partial^2 y}{\partial t^2} = \frac{B}{\rho}\frac{\partial^2 y}{\partial x^2}.$$

This is once again the wave equation for displacement. Traveling wave solutions of this equation look like $f(x \pm vt)$, where $v = (B/\rho)^{1/2}$. The main physical idea here is that a change in pressure produces a change in volume of the element. Since the change in volume is related to the displacement, the pressure and displacement are related through the bulk modulus.

4. Sinusoidal Waves

For all sinusoidal waves, there is a relationship between speed v, wavelength λ, frequency ν, angular frequency ω, and wave number k:

$$v = \lambda\nu = (\lambda/2\pi)(2\pi\nu) = \omega/k.$$

For nonsinusoidal waves it is not always possible to define a single, precise wavelength or frequency. You will be concerned in this module with sinusoidal waves.

5. Wave Movement

As a wave progresses through space, the quantity that varies (the "thing that's waving") does not move along with the wave. For example, in a traveling wave on a string, a particle on the string moves perpendicular to the string, while the wave moves along the string. In a sound wave, the individual molecules of the medium execute microscopic, longitudinal oscillations about an equilibrium position, while the wave moves on through the medium.

6. Energy and Momentum

Mechanical and electromagnetic waves therefore do not carry matter, but they do carry energy and linear momentum. (They may also carry angular momentum.) The intensity of a wave is the energy carried across unit area in a unit of time. The units of intensity are joules per second per square meter or watts per square meter. The brightness of a light, the loudness of a sound, the destructiveness of a "tidal wave" or tsunami, are all related to the intensity of the wave. The intensity is proportional to the square of the amplitude of a wave. For example, in Traveling Waves, you learned that the intensity of a wave on a string is proportional to the square of the transverse displacement. The intensity of a sound wave is proportional to the square of the pressure excursion, or to the square of the longitudinal displacement from the equilibrium position. Normally the intensity, or the average intensity, is the quantity that is measured, rather than the pressure amplitude, or the particle displacement.

ADDITIONAL LEARNING MATERIALS

Films

Superposition of Pulses in a Spring, S-81293, Encyclopaedia Britannica Educational Corporation.

Longitudinal Standing Waves in a Spring, S-81298, Encyclopaedia Educational Corporation.

Standing Sound Waves, S-81299, Encyclopaedia Educational Corporation.

Experiments in Physics (24): Standing Acoustic Waves, de Gruyter.

TEXT: Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

Keep this study guide readily available as you study. The comments on the material in the text are intended to direct your attention as you read, and thereby make your reading easier. Complete the entire study procedure before attempting the Practice Test.

For Objective 1, study Section 29.7 in Chapter 29. Recall from the module Traveling Waves that the amplitude of any wave is the maximum value of the oscillating quantity. Thus, ξ_0 in Eq. (29.12) is the displacement amplitude of the sound wave. Study Section 29.8 up to the last paragraph on p. 576. and General Comment 3.

Equation (29.14) is the wave equation. Every kind of wave (displacement of strings, light, water, sound) is described by a differential equation that is mathematically similar to Eq. (29.14). You need not remember the equation. It is enough, for now, if you can accept the fact that the solutions of Eq. (29.14) are superpositions of functions like

$$\xi(x, t) = \xi(x - vt) \quad \text{and} \quad \xi(x, t) = \xi(x + vt),$$

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Objective Number	Readings	Problems with Solutions	Assigned Problems		Additional Problems
		Study Guide	Study Guide	Text[a]	
1	Secs. 29.7, 29.8				Chap. 29, Quest. 8, 9
2	General Comments 1, 3, 4, 5, 6, Sec. 29.7	A	D	Chap. 29, Probs. 2, 6	Chap. 29, Probs. 3, 13, 20
3	Secs. 31.1 to 31.3, General Comment 2	B	E	Chap. 31, Probs. 1, 2, 7	Chap. 31, Probs. 3, 4, 8
4	Secs. 34.5, 34.6	C		Chap. 34, Quest. 6, 7, 11, Prob. 6	
5	Sec. 29.9		E	Chap. 29, Prob. 17	

[a]Quest. = Question(s).

where, from Eq. (29.15), $v = \sqrt{E/\rho}$. Particularly useful solutions of this type are of the form in Eq. (29.12). Solve Problems A and D in this study guide, and Problems 2 and 6 in Chapter 29 for Objective 2.

Read Section 31.1 in Chapter 31 for Objective 3, concentrating on the statement of the superposition principle, and General Comment 2. Then study Sections 31.2 and 31.3. For your own satisfaction, prove the trigonometric identity in the middle of p. 612. The easiest procedure is to begin by applying the sum and difference formulas to the right-hand side. Notice that the second equation from the bottom of p. 612 should, strictly speaking, be a proportionality instead of an equality. The constant of proportionality will, of course, have appropriate dimensions so that both sides have the units of intensity (joules per square meter per second)! The important point of these sections is that the principle of superposition tells us to add the amplitudes of interfering waves. However, if you were to detect the sound with a microphone, for example, the signal you received would be proportional to the average intensity rather than to the amplitude. Solve Problem B in this study guide, and Problems 1, 2, and 7 of Chapter 31.

Study Section 29.9 in Chapter 29. Notice that a source that is moving with respect to the medium produces sound with a wavelength that differs from the wavelength due to a source at rest with respect to the medium. If the source is at rest, but the observer is moving, the wavelength remains unchanged, of course, but the observer intercepts a different number of wavelengths per unit time, and therefore perceives a different frequency. Solve Problem E in this study guide, and Problems 16 and 17 in Chapter 29.

Study Sections 34.5 and 34.6 in Chapter 34. Bueche introduces this material on resonance late in his text. He is therefore able to use words like "quantum" and "eigenfrequencies," which you may not have learned. Don't worry about it, since in the context of Section 34.6, you may cross out the words "quantum" and the prefix "eigen-," and not change the meaning at all. Solve Problem C in this study guide. Answer Questions 6 and 7 and solve Problem 6 in Chapter 34.

STUDY GUIDE: Sound 8(HR 1)

Text: David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974)

SUGGESTED STUDY PROCEDURE

Keep this study guide open and readily available as you read. The comments on material in the textbook are intended to direct your attention as you read, and thereby make your reading easier. Complete the entire study procedure before attempting the Practice Test.

Read quickly Section 17-1 in Chapter 17. Study Section 17-2 (except for the final two paragraphs), and General Comment 3. Study Section 17-3. In Eq. (17-2), note that if displacement (pressure) is represented by such an expression, pressure (displacement) is represented by a similar expression, 90° out of phase with displacement (pressure). Solve Problems A and D in this study guide, and Problem 11.

Study (or review) Section 16-7 and General Comment 2. The principle of superposition is applied to a very large variety of situations. Solve Problem B in this study guide, and Problems 27 and 29 in Chapter 16. Then study Section 17-4 and solve Problem C in this study guide, and Problems 25, 30, and 31 in Chapter 17. Study Sections 17-5 and 17-6 and solve Problem E in this study guide, and Problems 42, 43, and 44 in Chapter 17.

HALLIDAY AND RESNICK

Objective Number	Readings	Problems with Solutions	Assigned Problems		Additional Problems
		Study Guide	Study Guide	Text	
1	Secs. 17-1 to 17-3				
2	Sec. 17-3, General Comments	A	D	Chap. 17, Prob. 11	
3	Secs. 16-7, 17-5, General Comment 2	B	E	Chap. 16, Probs. 27, 29	
4	Sec. 17-4	C			Chap 17, Probs. 37, 38
5	Sec. 17-6		E	Chap. 17, Probs. 42, 43, 44	Chap. 17, Probs. 41, 45, 46

STUDY GUIDE: Sound 8(SZ 1)

TEXT: Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition

SUGGESTED STUDY PROCEDURE

Keep this study guide open and readily available as you read. The comments on material in the text are intended to direct your attention as you read, and thereby make your reading easier. Complete the entire study procedure before attempting the Practice Test.

Review Sections 21-1 and 21-2 in Chapter 21. Concentrate especially on the nature of a longitudinal wave, and the mathematical form of a traveling wave, Eq. (21-3). The wave equation is at the end of Section 21-2. Read the General Comments.

Solve Problems A and D in this study guide, and Problem 21-3 in the text. This deals with a transverse wave, but you may pretend it is a longitudinal displacement. Study Section 21-4, then read Sections 23-1 through 23-4 for a qualitative discussion of some aspects of sound and music.

Study Sections 22-6 through 22-8, and General Comment 2. Your text distinguishes between reinforcement, when waves add constructively, and interference,

SEARS AND ZEMANSKY

Objective Number	Readings	Problems with Solutions	Assigned Problems		Additional Problems
		Study Guide	Study Guide	Text	
1	Secs. 21-1, 21-4, 23-1, 23-2				
2	Sec. 21-2, General Comments	A	D	21-3	
3	Secs. 22-2, 22-6, 22-7, 23-8, General Comment 2	B	E	23-7(a)	23-15
4	Sec. 22-8	C			
5	Sec. 23-9		E	23-14, 23-16, 23-17	23-11, 23-12, 23-13, 23-15

when waves add destructively. More common usage of the words is to refer to both of these effects as interference. Solve Problems B and C in this study guide, and Problem 23-7(a) in your text.

Study Section 23-8, which describes another important application of the principle of superposition. For your own satisfaction, prove the trigonometric identity just before Eq. (23-6). The easiest procedure is to apply the well-known formulas for the cosine of the sum or difference of two angles to the right-hand side. Study Section 23-9, and solve Problem E in this study guide, and Problems 23-14, 23-16, and 23-17 in the text.

STUDY GUIDE: Sound 8(WS 1)

TEXT: Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 1

SUGGESTED STUDY PROCEDURE

Keep this study guide open and readily available as you study. The comments on the material in the text are intended to direct your attention as you read, and thereby make your studying easier. Complete the entire study procedure before attempting the Practice Test.

Study carefully Sections 17-1 and 17-2 in Chapter 17. If you have not studied thermodynamics, omit the final four paragraphs of Section 17-2. Study Section 17-3 and the General Comments. Solve Problems A and D in this study guide, and Problems 17-3, 17-8, and 17-9 in the text. Study Section 17-4. It is characteristic of virtually all waves that the intensity is proportional to the square of the amplitude. The intensity, averaged over many oscillations, is usually the quantity that is measured.

Study Section 17-5. You may also wish to review Sections 16-6 and 16-7. Although these sections are concerned with transverse waves instead of longitudinal waves, the analysis of superposition phenomena is identical to that for longitudinal waves. Solve Problems B and C in this study guide, and Problems 17-12, 17-13, and 17-14 in the text. Study General Comment 2. Read Section 17-7, and study Section 17-11. Solve Example 17-5 and Problem E in this study guide. Then solve Problems 17-22 and 17-25.

WEIDNER AND SELLS

Objective Number	Readings	Problems with Solutions	Assigned Problems		Additional Problems
		Study Guide	Study Guide	Text	
1	Secs. 17-1 to 17-4, 17-7				
2	Secs. 17-1, 17-3, General Comments	A	D	17-3, 17-8, 17-9	17-4, 17-5
3	Secs. 16-6, 16-7, 17-5, General Comment 2	B	E		
4	Sec. 17-5	C		17-12, 17-13, 17-14	
5	Sec. 17-11		E	17-22, 17-25	17-21, 17-24

PROBLEM SET WITH SOLUTIONS

A(2). The deviation of the air pressure from its equilibrium value in a certain region is given by the expression $p(x, t) = 20 \sin(6x - 6000t)$ N m^2. (All units are SI.) Does this expression represent a traveling wave in air?

Solution

Solutions of the wave equation can be expressed as a function of $(x \mp vt)$. If we can put the given expression in that form, we shall be able to determine the answer to the question:

$$p(x, t) = 20 \sin(6x - 6000t) = 20 \sin 6(x - 1000t).$$

By comparing with $p(x - vt)$, we immediately see that the given expression represents a wave traveling in the direction of increasing x, with a speed of 1000 m/s, and a pressure amplitude of 20.0 N/m^2. It may represent a sound wave, but not a sound wave in air at STP, where v = 330 m/s.

Although it was not part of the original question, it's good practice to investigate the properties of the wave a little further. For example, the wave number k is 6 m^{-1}, thus the wavelength is

$$\lambda = 2\pi/k = 6.28/6 = 1.05 \text{ m}.$$

Since ω is 6000 rad/s, the frequency is

$$\nu = \omega/2\pi = 6000/6.28 = 955 \text{ Hz},$$

a value in the audible range.

B(3). Two sources of plane waves, S_1 and S_2, are located distances ℓ_1 and ℓ_2 from an observer P, as shown in Figure 4. The sources broadcast in phase with each other. The pressure at either source is

$$p_s = p_0 \cos(\omega t).$$

Calculate the intensity of sound measured by the observer at P relative to that observed if only one of the sources is working.

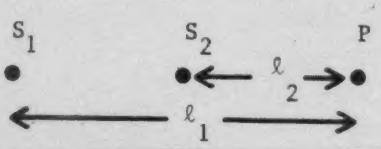


Figure 4

Solution

The principle of superposition tells us to add pressures:

$$P_p = p_0 \cos(k\ell_1 - \omega t) + p_0 \cos(k\ell_2 - \omega t).$$

From the trigonometric identity,

$$\cos a + \cos b = 2 \cos[(a - b)/2] \cos[(a + b)/2],$$

thus

$$\begin{aligned} P_p &= 2p_0 \cos[(k\ell_1 - \omega t - k\ell_2 + \omega t)/2] \cos[(k\ell_1 + k\ell_2 - \omega t + k\ell_2 - \omega t)/2] \\ &= 2p_0 \cos[(k\ell_1 - k\ell_2)/2] \cos[k\ell_1 + k\ell_2 - 2\omega t)/2], \end{aligned}$$

where $k(\ell_1 - \ell_2) = (2\pi/\lambda)\Delta\ell$ is the relative phase of the sound from the two sources. Average intensity is proportional to square amplitude, thus

$$Ip \propto 4p_0^2 \cos^2[(k/2)(\ell_1 - \ell_2)].$$

Thus the relative intensity is

$$I_p/I_1 = 4 \cos^2[(k/2)(\Delta\ell)].$$

If $k\,\Delta\ell/2 = (2n + 1)(\pi/2)$, where n = integer, the intensity is always zero. If $k\,\Delta\ell/2 = n\pi$, the intensity will be maximum and will be four times as large as I_1.

C(4). A tuning fork is placed over a vertical tube that is open at the top and can be filled with water. It produces strong resonances when the water is 0.080 m and 0.280 m from the top of the tube, and under no other conditions. What is the frequency of the tuning fork? The speed of sound in air is 330 m/s.

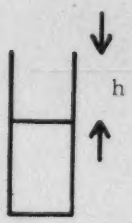


Figure 5

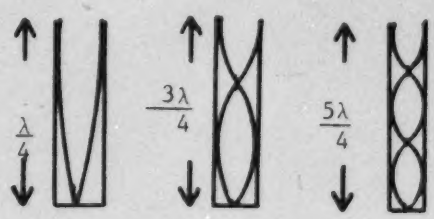


Figure 6

Solution

The water surface closes off the lower end of the tube. See Figure 5. The water presents an immovable barrier to the displacement of the air molecules, thus there is a displacement node at the water surface. Near the top of the tube, on the other hand, there is a displacement antinode. See Figure 6. The top of the tube and the water must therefore be (2n - 1)/4 wavelengths apart.

We have

$$h_1 = [(2n_1 - 1)/4]\lambda \quad \text{and} \quad h_2 = [(2n_2 - 1)/4]\lambda,$$

where h_1 is the minimum length to produce a resonance, thus $n_1 = 1$. The next highest must be $n_2 = 2$:

$$h_2 - h_1 = \left(\frac{2n_2 - 1}{4} - \frac{2n_1 - 1}{4}\right)\lambda = \frac{n_2 - n_1}{2}\lambda = \left(\frac{n_2 - n_1}{2}\right)\left(\frac{v}{\nu}\right),$$

$$\nu = \left(\frac{n_2 - n_1}{2}\right)\left(\frac{v}{h_2 - h_1}\right) = \left(\frac{1}{2}\right)\left(\frac{300}{0.200}\right) = 825 \text{ Hz}.$$

Problems

D(2). The displacement of a particle in a medium is given by the expression

$$y = \{(1.00 \times 10^{-10}) \exp[-(5x)^2] \exp[-(1650t)^2] \exp[-15(1650)xt]\} \text{ m}.$$

Does this represent a traveling wave?

E(3, 5). Maurice André stands beside a railroad track with his trumpet. Dizzy Gillespie stands on a flatcar with his trumpet. The flat car rolls toward André at a speed $v_G = 3.30$ m/s. Each man plays a concert A (440 Hz). How many beats per second do you hear if you are standing on the track between them? The speed of sound in the still air is 330 m/s.

Solutions

D(2). Let us change the product of exponentials to a sum of exponents, and see if $y = y(x \mp vt)$:

$$y = 10^{-10} \exp[-(5x)^2 - (1650t)^2 - 15(1650)xt].$$

Does the exponent factor?

$$(5x)^2 + (1650t)^2 + 15(1650)xt = (5)^2[x^2 + 3(\frac{1650}{5})xt + (\frac{1650}{5})^2 t^2].$$

The quantity in brackets is <u>almost</u> a perfect square, but not quite. <u>If</u> the 3 had been a 2, the exponent would have been

$$-(5)^2[x^2 + 2(\frac{1650}{5})xt + (\frac{1650t}{5})^2] = -(5)^2(x + \frac{1650}{5}t)^2.$$

This expression represents a wave, moving in the direction of <u>decreasing</u> x (why?), with

$$v = 1650/5 = 330 \text{ m/s}.$$

This is close to the speed of sound in air. However, the problem as given cannot be put in the form $y(x, t) = y(x + vt)$, and therefore does not represent a simple traveling wave.

E(3, 5). Because the flatcar is moving toward the observer the wavelength of Gillespie's tone will be shortened. At rest, the distance between crests of the sound wave would be $\lambda_0 = v/\nu_0$. However, in one period of the sound, a time $1/\nu_0$ s, Diz advances a distance $v_G(1/\nu_0)$, thus the new wavelength is

$$\lambda = \lambda_0 - \frac{v_G}{\nu_0} = \frac{v - v_G}{\nu_0}, \qquad \frac{1}{\lambda} = \frac{\nu_0}{v - v_G},$$

$$\nu = \frac{v}{\lambda} = \nu_0\left(\frac{v}{v - v_G}\right) = \nu_0\left(\frac{1}{1 - v_G/v}\right).$$

The frequency from Diz's trumpet is

$$\nu = 440(1/1 - 0.0100) \simeq 444 \text{ Hz}.$$

If you remember that the number of beats between two sounds is equal to the difference in frequencies, you can immediately say that the number of beats is

$$\Delta\nu = 444 - 440 = 4 \text{ beats/s}.$$

(On a test, you might be asked to give at least a qualitative explanation of that answer.)

PRACTICE TEST

Useful information: the speed of sound in air at STP is v = 330 m/s,

$\cos a + \cos B = 2 \cos(\frac{a - b}{2}) \cos(\frac{a + b}{2})$; $\sin a + \sin b = 2 \cos(\frac{a - b}{2}) \sin(\frac{a + b}{2})$.

1. Explain, in your own words, what sound is. Define the displacement amplitude, pressure amplitude, and intensity. How are they related? (On the Mastery Test, your instructor will tell you whether to answer this question in writing or orally.)

2. Does the following expression for pressure represent a possible sound wave? If so, determine the amplitude of the wave, and the velocity of sound in the medium. If not, state how you know that it is not a sound wave:

$$p(x, t) = 2.00 \times 10^{-3} \sin(0.50x - 250t) + 3.50 \times 10^{-4} \cos(2.50x + 1250t).$$

3. The fundamental tone of a pipe that is closed on one end is a concert A (440 Hz) when the tone is excited in air at STP. (a) Determine the length of the pipe. (b) How fast must I move if I wish to raise the tone I hear from the pipe to a B-flat (466 Hz)? Should I move toward or away from the pipe?

4. Two adjacent sources emit sound of equal intensity, and of frequencies 256 and 384 Hz, respectively. Calculate the beat frequency when these sources sound simultaneously. Can you interpret your answer physically?

Practice Test Answers

1. Sound is a longitudinal mechanical wave, propagated through a medium. The independent variable can be considered either the deviation of the pressure from its equilibrium value, or the displacement of particles of the medium from their equilibrium positions. The amplitude of the wave is then the maximum excursion of the pressure from its equilibrium value, or the maximum displacement. An increase in pressure will produce a decrease in the volume of an element of the medium. For a plane wave, the volume of an element with sides parallel to the direction of propagation is A Δx, where A is the cross-sectional area and Δx is the thickness of the element. Thus, the displacement is directly related to the pressure excursion. This answer is adequate. A more quantitative description of the relation between pressure and displacement involves the bulk modulus B: $p = -BkY_0 \cos(kx - \omega t)$.

2. The expression represents the superposition of two waves traveling at v = 500 m/s. Notice that if the two speeds were different, the expression would not represent a wave, since the speed of sound is independent of the frequency.

3. (a) $\lambda = v/4\upsilon = 0.187$ m. (b) $v_1 = v(\upsilon'/\upsilon - 1) = 19.8$ m/s^2. Toward.

4. $\Delta\upsilon = 128$ Hz. This frequency is too high to be heard as "beats." It is the frequency of C an octave below middle C. If you play middle C and the G above it loudly enough, you will hear the lower C.

SOUND

Date _____

Mastery Test Form A

pass recycle

1 2 3 4 5

Name _____

Tutor _____

For the speed of sound in air at STP, use v = 330 m/s.

$\cos a + \cos b = 2 \cos(\frac{a - b}{2}) \cos(\frac{a + b}{2})$; $\sin a + \sin b = 2 \cos(\frac{a - b}{2}) \sin(\frac{a + b}{2})$.

1. Explain, in your own words, what sound is. Define the displacement amplitude, pressure amplitude, and intensity. How are they related?

2. State whether the expression

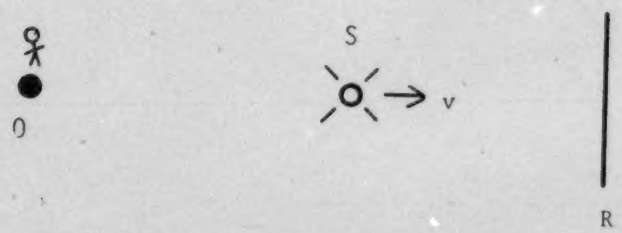
$$p = 20.0 \sin(337t + 0.876x) \text{ N/m}^2$$

represents a traveling wave. If it does, what is the speed of sound? Name a point on the x axis where the excess pressure is zero at t = 0. Calculate the distance to the nearest position at which $p = p_{max} = 20.0 \text{ N/m}^2$ at t = 0.

3. A sound source S, emitting a frequency of 500 Hz, moves toward a reflector R at 30.0 m/s as in Figure 1. An observer O hears sound from S and from R. What beat frequency does he hear?

4. A pipe open on each end has a fundamental frequency of 330 Hz. The first overtone of the pipe has the same frequency as the first overtone of another pipe closed on one end. Calculate the length of each pipe.

Figure 1



SOUND

Mastery Test Form B

Date _____

pass recycle

1 2 3 4 5

Name _____ Tutor _____

For the speed of sound in air at STP, use v = 330 m/s.

$\cos a + \cos b = 2 \cos(\frac{a - b}{2}) \cos(\frac{a + b}{2})$; $\sin a + \sin b = 2 \cos(\frac{a - b}{2}) \sin(\frac{a + b}{2})$.

1. Explain, in your own words, what sound is. Define the displacement amplitude, pressure amplitude, and intensity. How are they related?

2. If the expression

$$\xi = \xi_0 \exp(-3x^2) \exp(+1980xt) \exp(-3.267 \times 10^5 t^2) \sin(\sqrt{3}x - \sqrt{3}(330)t)$$

can represent the displacement in a traveling sound wave, sketch the wave as a function of x at t = 0. If it cannot represent a wave, explain why.

3. A man moves between two identical whistles. When his speed is 20.0 m/s, he hears a beat frequency of 30.0 Hz. What is the frequency of the sound emitted by the whistles?

4. Two pipes, a and b, have the same fundamental frequency. Pipe a is open on each end, and pipe b is closed on one end.

   (a) Which pipe is longer?

   (b) Compare the frequencies of overtones from the two pipes.

SOUND

Date _____

Mastery Test Form C

pass recycle

1 2 3 4 5

Name _____

Tutor _____

For the speed of sound in air at STP, use v = 330 m/s.

$\cos a + \cos B = 2 \cos(\frac{a - b}{2}) \cos(\frac{a + b}{2})$; $\sin a + \sin b = 2 \cos(\frac{a - b}{2}) \sin(\frac{a + b}{2})$.

1. Explain, in your own words, what sound is. Define the displacement amplitude, pressure amplitude, and intensity. How are they related?

2. A purported wave is represented by the expression

$$p = 0.150 \cos(2.00x + 1000t) + 0.0300 \sin(500t - 1.00x).$$

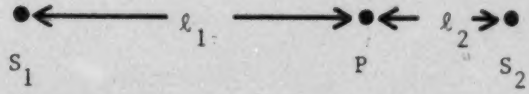
Is this expression, in fact, a possible sound wave? If so, calculate the speed of sound. If not, explain why not.

3. Two identical sources of sound, S_1 and S_2, are located distances ℓ_1 and ℓ_2 from an observer P, as shown in Figure 1. The sources broadcast plane waves in phase with each other. The displacement amplitude at each source is $\xi = \xi_0 \cos \omega t$.

(a) Calculate the intensity of sound measured by the observer, relative to the intensity when only S_2 is operating.

(b) Choose some "reasonable" numbers that will maximize the intensity at P.

Figure 1



4. A tuning fork is held over a tube partially filled with water. Resonance occurs when the water is 0.100 and 0.260 m below the fork. Calculate the frequency of the tuning fork.

5. A jet aircraft moves at a speed three-fourths that of sound. It overtakes another aircraft moving at only one-fourth that of sound. The jet is emitting a sound with the frequency 280 Hz. What frequency does a passenger on the slower aircraft hear before the jet passes?

MASTERY TEST GRADING KEY - Form A

1. What To Look For: Longitudinal wave in pressure or displacement from equilibrium value. Physical argument relating pressure and displacement. Analytic treatment, using bulk modulus, is OK, but not required. Proportionality between I and (pressure)^2 or (displacement)^2.

   Solution: Sound is a longitudinal mechanical wave, propagated through a medium. The independent variable can be considered either the deviation of the pressure from its equilibrium value, or the displacement of particles of the medium from their equilibrium positions. The amplitude of the wave is then the maximum excursion of the pressure from its equilibrium value, or the maximum displacement. An increase in pressure will produce a decrease in the volume of an element of the medium. For a plane wave, the volume of an element with sides parallel to the direction of propagation is A Δx, where A is the cross-sectional area and Δx is the thickness of the element. Thus, the displacement is directly related to the pressure excursion. This answer is adequate. A more quantitative description of the relation between pressure and displacement involves the bulk modulus B: $p = -Bky_0 \cos(kx - \omega t)$.

2. What To Look For: Wave must be expressed as $f(x \mp vt)$. Student should show ability to pick a particular time and evaluate what happens.

   Solution:

   $p = 20.0 \sin(377t + 0.876x) = 20.0 \sin 0.876[x + (377/0.876)t]$.

   This is a function of $(x + vt)$, thus it is a sound wave, with speed $377/0.876 = 430$ m/s.

   $p(x, 0) = 0 = 20.0 \sin(0.876x)$, $\quad 0.876x = n\pi$, $\quad n = 1, 2, \ldots$.

   $x = n\pi/0.876 = n(3.59)$ m. Nearest maximum is $\lambda/4$ away:

   $\lambda/4 = 2\pi/4k = \pi/2(0.876) = 1.793$ m.

3. What To Look For: Direction (+) of frequency shift should be correct. Use of correct equations, and ability to justify physically the choice of equations. Can he justify physically the fact that beat frequency is ν? (Modulation of amplitude is $\Delta\nu/2$, but modulation of intensity is $\Delta\nu$.)

   Solution: Source emits ν wavelengths. Source moves (v_s/ν) m_1 in one period. Wavelength of sound is $\lambda' = \lambda_0 \mp v_s/\nu$ for waves emitted to [right, left]. The corresponding frequencies are

$$\nu = \frac{c}{\lambda'} = \frac{c}{\lambda_0 \mp v_s/\nu} = \frac{c\nu}{c \mp v}.$$

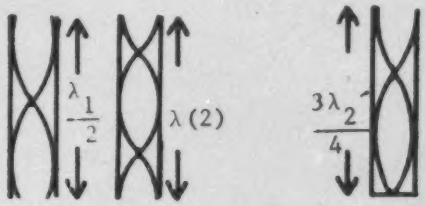
   Observer receives both frequencies, so the beat frequency is

$$\Delta\nu = \nu c\frac{1}{c - v} - \frac{1}{c + v} = \frac{2\nu vc}{c^2 - v^2} = \frac{2\nu vc}{(c - v)(c + v)} = \frac{2(500)(30)(330)}{(300)(360)} = \frac{11(10^3)}{10(12)}$$

$\Delta\nu$ = 91.7 Hz.

4. What To Look For: How displacement and pressure depend on configuration of pipe. Ability to connect conditions. Ask about ν_2.

Figure 9



Solution: See Figure 9, in which the first two sketches show displacement amplitude for fundamental (λ_1) and first overtone (λ_2) in open pipe. Third sketch shows first overtone (λ_2') of second pipe. Let $\ell + \ell'$ be length of open and closed pipes:

$$\ell = \ell/2 = c/2\nu_1,$$

where $\nu_1 = 330$, $\ell = \lambda_2 = c/\nu_2$, $\ell' = 3\lambda_2'/4 = 3c/4\nu_2'$. From the first equation, $\ell = 330/2(330) = (1/2)$ m. From the second equation, $\nu_2 = c/\ell = 330(2) = 660$. That makes sense! From the third equation, $\ell' = 3(330)/4(660) = (3/8)$ m.

MASTERY TEST GRADING KEY - Form B

1. Solution: Sound is a longitudinal mechanical wave, propagated through a medium. The independent variable can be considered either the deviation of the pressure from its equilibrium value, or the displacement of particles of the medium from their equilibrium positions. The amplitude of the wave is then the maximum excursion of the pressure from its equilibrium value, or the maximum displacement. An increase in pressure will produce a decrease in the volume of an element of the medium. For a plane wave, the volume of an element with sides parallel to the direction of propagation is A Δx, where A is the cross-sectional area and Δx is the thickness of the element. Thus, the displacement is directly related to the pressure excursion. This answer is adequate. A more quantitative description of the relation between pressure and displacement involves the bulk modulus B: $p = -Bky_0 \cos(kx - \omega t)$.

2. What To Look For: Demonstrate that student should look for $f(x \mp vt)$.

   Solution:

   $\xi = \xi_0 \exp(-3x^2) \exp(1980xt) \exp(-3.267 \times 10^5 t^2) \sin[\sqrt{3}x - \sqrt{3}(330)t]$.

   Exponent is $-3[x^2 - 660xt + 1.089 \times 10^5 t^2] = -3(x - 330t)^2$.

   $\xi = \xi_0 \exp[-(kx - \omega t)^2 \sin(kx - \omega t)$,

   where $k = \sqrt{3}\ m^{-1}$; $\omega = vk = \sqrt{3}(330)$ rad/s, and V = 330 m/s. It <u>is</u> a wave.

3. What To Look For: Does wavelength change? (No.) Student should know which has highest frequency.

   Solution: See Figure 10. In 1 s, man moves v_0 m, or v_0/λ wavelengths. Number of wavelengths he intercepts is $\nu \pm v_0/\lambda$, depending on whether he moves toward or away from the source. The difference in frequency is $2(v_0/\lambda)$, which is the beat frequency:

   $$2\left(\frac{v_0}{\lambda}\right) = 2\left(\frac{v_0 \nu_0}{c}\right) = 30, \qquad \nu_0 = \frac{30(330)}{2(20)} = \frac{3}{4}(330) = 247.5 \text{ Hz}.$$

Figure 10

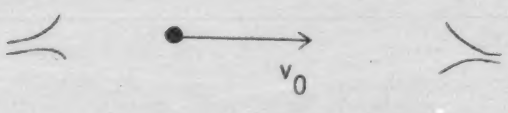
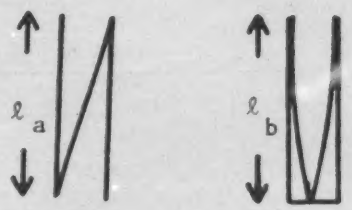


Figure 11



4. What To Look For: Know conditions for resonance. Be able to relate conditions in the two pipes.

Solution: See Figure 11. (a) Displacement distribution is shown in Figure 11:

$$\ell_a = \lambda/2, \quad \ell_b = \lambda/4,$$

(b) For pipe a,

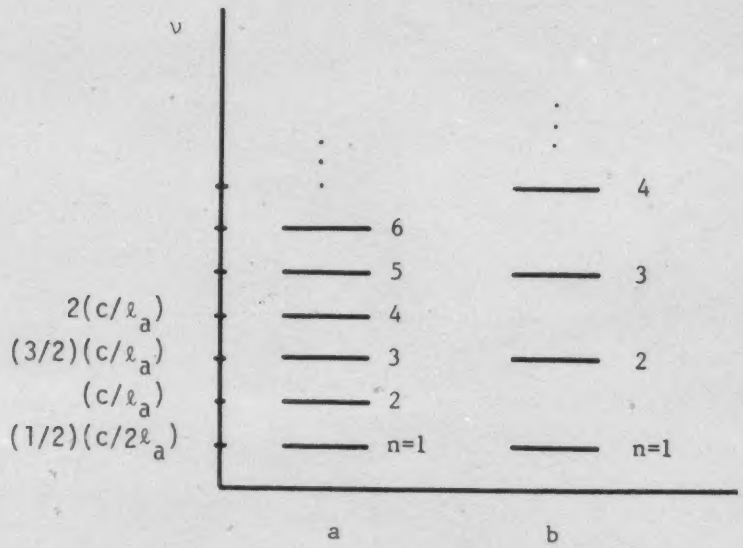
$$\ell_a = \frac{n\lambda_n^{(a)}}{2} = \frac{nc}{2\nu_n^{(a)}}, \quad \nu_n^{(a)} = \frac{nc}{2\ell_a}, \quad n = 1, 2, \ldots.$$

For pipe b,

$$\ell_b = (2n - 1)\frac{\lambda_n^{(b)}}{4} = \frac{(2n - 1)c}{4\nu_n^{(b)}}, \quad \nu_n^{(b)} = \frac{(2n - 1)c}{4\ell_b}.$$

For n = 1, we know the two frequencies are the same: $\nu_1^{(a)} = \nu_1^{(b)}$. See Figure 12.

Figure 12



MASTERY TEST GRADING KEY - Form C

1. Solution: Sound is a longitudinal mechanical wave, propagated through a medium. The independent variable can be considered either the deviation of the pressure from its equilibrium value, or the displacement of particles of the medium from their equilibrium positions. The amplitude of the wave is then the maximum excursion of the pressure from its equilibrium value, or the maximum displacement. An increase in pressure will produce a decrease in the volume of an element of the medium. For a plane wave, the volume of an element with sides parallel to the direction of propagation is $A\,\Delta x$, where A is the cross-sectional area and Δx is the thickness of the element. Thus, the displacement is directly related to the pressure excursion. This answer is adequate. A more quantitative description of the relation between pressure and displacement involves the bulk modulus B: $p = -Bky_0 \cos(kx - \omega t)$.

2. What To Look For: Student should know that wave is of the form $f(x + vt)$. Ask student whether it would be a sound wave if one of the ω's were changed.

   Solution:

   $$p = 0.150 \cos(2.00x + 1000t) + 0.0300 \sin(500t - 1.00x)$$
   $$= 0.150 \cos[2(x + 500t)] + 0.0300 \sin[-(x - 500t)].$$

   It is a sound wave, with v = 500 m/s.

3. Solution: Choose origin of coordinate system at S_1 as in Figure 13. Then

   $$\xi_1(x, t) = \xi_0 \cos(kx - \omega t), \quad \xi_2(x, t) = \xi_0 \cos[k(x - \ell_1 - \ell_2) - \omega t].$$

   At point P,

   $$\xi(\ell_1, t) = \xi_0 \cos(k\ell_1 - \omega t) + \xi_0 \cos(-k\ell_2 - \omega t)$$
   $$= 2\xi_0 \cos\{[k(\ell_1 - \ell_2) + 2\omega t]/2\} \cos[k(\ell_1 + \ell_2)/2]$$
   $$= 2\xi_0 \cos[k(\ell_1 + \ell_2)/2]\{\cos[k(\ell_1 - \ell_2)/2] \cos \omega t$$
   $$- \sin[k(\ell_1 - \ell_2)/2] \sin \omega t\}$$

Figure 13

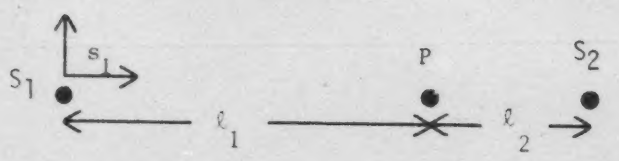
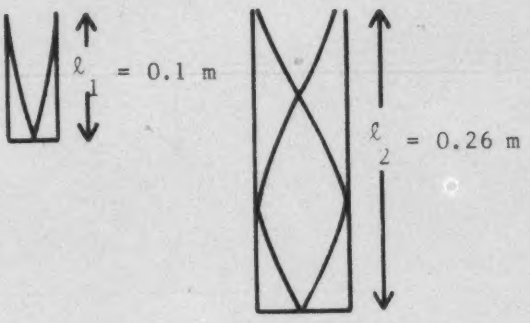


Figure 14



For a maximum, we clearly need

$\frac{k(\ell_1 + \ell_2)}{2} = n_1\pi$, $n = 0, 1, 2, \ldots$ and $\frac{k(\ell_1 - \ell_2)}{2} = n_2\pi$,

or $\frac{k(\ell_1 - \ell_2)}{2} = (2n_2 + 1)\frac{\pi}{2}$.

Let us choose

$k(\ell_1 + \ell_2)/2 = n_1\pi$, $k(\ell_1 - \ell_2)/2 = n_2\pi$.

Then $k\ell_1 = (n_1 + n_2)\pi$ and $k\ell_2 = (n_1 - n_2)\pi$. Let $\ell_1 = \ell_2$. Then $n_2 = 0$, and choose

$k_2(\frac{2\ell}{2}) = n_1\pi$, $\ell = \frac{n_1\pi}{k} = \frac{n_1\lambda}{2} = \frac{n_1 c}{2\nu}$. If we choose ν as, say, 330 Hz, then $\ell = (1/2)$ m.

4. What To Look For: Student should know and apply condition for resonance.

Solution: See Figure 14.

$\ell_1 = \frac{\lambda}{4} = \frac{c}{4\nu}$, $\ell_2 = \frac{3\lambda}{4} = \frac{3c}{4\nu}$, $\ell_2 - \ell_1 = \frac{c}{4\nu}(3 - 1) = \frac{c}{2\nu}$,

$\nu = \frac{c}{2(\ell_1 - \ell_2)} = \frac{330}{2(0.320)} = 516$ Hz.

5. What To Look For: Student should distinguish between motion of source and receiver and use appropriate formulation.

Solution: In one period $T = 1/\nu_0$, the fast jet advances

$\Delta\ell = v_1 T = \nu_1/\nu_0 = (3/4)(c/\nu_0)$.

Wavelength is shortened by $\lambda_1 = \lambda_0 - 3c/4\nu_0 = \lambda_0/4$. The slow craft, in one second, receives $\nu_1 - \nu_2/\lambda_1$ wave crests so

$\nu_2 = \nu_1 - v_2/\lambda_1 = 4\nu_0 - (c/4)(\nu_1/c) = 4\nu_0 - (4\nu_0/4) = 3\nu_0$.

Thus $\nu_2 = 3(280) = 840$ Hz.

STUDY GUIDE

TEMPERATURE, HEAT, AND THERMODYNAMICS: First Law

INTRODUCTION

How can we keep track of energy as it is transferred from one system to another? How can we calculate the amount of internal energy - a quantity that seems to be hidden within the very "guts" of matter? Further, what is the difference between temperature and heat, and between heat and work?

This module focuses on the first of two central thermodynamic principles: the conservation of energy, or, as it is sometimes called, the first law of thermodynamics. The second basic principle, which deals with the inevitable increase of a quantity called entropy, is the subject of another module Second Law and Entropy. These two abstract principles, plus a few other concepts and laws and the vocabulary needed for literacy in the field, are the entire content of thermodynamics. The energy and entropy principles form the framework that governs all energy conversions involving heat; they are the touchstones we must rely on as we attempt to create new energy devices, such as solar converters or fusion reactors, to limit the wasteful exploitation of the Earth's resources.

The approach of this module is macroscopic - that is, we shall deal with systems that are approximately of human scale in size and mass (thermometers, blocks of ice, heat engines), and we shall choose observable quantities such as pressure, volume, and temperature to describe the behavior of these systems. The macroscopic approach should be seen as supplementary to the microscopic approach, which regards the behavior of the atoms and molecules as fundamental. This latter framework chooses the molecular velocities, energies, and momenta as the starting point, and values for macroscopic observables are derived from the microscopic picture. The microscopic approach is treated in another module Kinetic Theory of Gases, where the behavior of gases is interpreted in terms of molecular energies and collisions.

PREREQUISITES

Before you begin this module, you should be able to:	Location of Prerequisite Content
*Interpret an integral as the area under a curve and evaluate it (needed for Objective 5 of this module)	Calculus Review
*Calculate the work done by a constant or variable force oriented parallel or obliquely to the displacement of a particle (needed for Objective 5 of this module)	Work and Energy Module
*Calculate the kinetic energy of a particle, given its mass and velocity [specifically, use the formula $K = (1/2)mv^2$] (needed for Objective 5 of this module)	Work and Energy Module
*Employ the relationship of power to energy and time ($E = Pt$) in order to calculate one of the quantities, given the others (needed for Objectives 3 and 5 of this module)	Work and Energy Module
*Calculate the amount of gravitational potential energy near the surface of the Earth (specifically, use the formula $U_g = mgh$) (needed for Objective 5 of this module)	Conservation of Energy Module

LEARNING OBJECTIVES

After you have mastered the content of this module, you will be able to:

1. Temperature and zeroth law - (a) State the zeroth law of thermodynamics and define the terms used; (b) deduce Kelvin temperature from the pressure of a gas thermometer; and (c) convert these temperatures from the Kelvin to the Celsius scale and vice versa.

2. Specific heat capacity - Define specific heat capacity, and solve mixture problems (in which several materials insulated from the environment and originally at different temperatures attain equilibrium).

3. Heat conduction - State the heat-conduction equation, define each term used, and, given a conductor of uniform cross section, find the value of whichever variable is unknown.

4. Latent heat and change of phase - Define latent heat, and solve problems in which a substance changes phase and/or temperature.

5. First law of thermodynamics - State the first law of thermodynamics, define the terms involved (heat, work, internal energy), and calculate these quantities for a system undergoing a given process, possibly presented in the form of a pressure-volume (pV) graph. The calculation may require you to convert kinetic or gravitational potential energy to internal energy.

STUDY GUIDE: Thermodynamics - First Law 3(B 1)

TEXT: Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

In contrast with the strictly macroscopic point of view in this module, Bueche explains thermodynamics largely within the framework of kinetic theory. To get an overview, we recommend that you first read fairly quickly through Sections 15.1 through 15.4 in Chapter 15, Sections 16.1 through 16.5 and 16.9 through 16.15 in Chapter 16, and Sections 17.1 through 17.4 in Chapter 17. Keep the macroscopic-microscopic distinction in mind; you will not be expected to learn any aspects of kinetic theory in this module.

For your second reading, we recommend that you proceed sequentially by objective, as listed in the Table. It will also be important to study the listed problems with solutions and to work the assigned problems. The table lists only the sections dealing with macroscopic thermodynamics, and in cases where both

BUECHE

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text	Study Guide	Text	
1	Sec. 15.3; second paragraph on p. 302 of Sec. 17.2; Quest. 4 of Chap. 17; Sec. 16.15; General Comment 1	A		G		
2	Secs. 16.1, 16.2, 16.3, General Comment 2	B	Illus.[a] 16.1	H		Chap. 16, Probs. 1, 3, 5, 6
3	Sec. 16.14	C	Illus. 16.8	I		Chap. 16, Probs. 24 to 26
4	Secs. 16.10, 16.12	D	Illus. 16.3 to 16.5	J		Chap. 16, Probs. 15 to 17
5	Secs. 16.5, 17.1, 17.2 (first paragraph of p. 301), 17.3 up to Eq. (17.3), first paragraph of Sec. 17.4, General Comments 3 to 9	E, F	Illus. 17.1	K, L	Chap. 17, Prob. 1	Chap. 17, Prob. 2

[a]Illus. = Illustration(s).

microscopic and macroscopic ideas are presented together, you should concentrate on the latter. If you wish to do further reading on macroscopic thermodynamics, an excellent treatment is in University Physics, Chapters 15 to 19.*

For Objective 1, read Section 15.3, the definition of thermodynamic equilibrium on p. 302 and the statement of the zeroth law of thermodynamics in Question 4 (p. 320) of Chapter 17, and Section 16.15. Read General Comment 1. Then work Problems A and G. Bueche defines a system to be in thermal equilibrium if the system is identical with a similar isolated system that has existed unchanged for an infinitely long time (p. 302). There are sound reasons behind such a definition, but it is difficult to apply in practice because of the infinite time required. We shall say that two systems are in thermal equilibrium when they are in thermal contact (for example, separated by a thin sheet of copper) and when macroscopic changes have ceased to occur. It is important to realize that the zeroth law ("two systems, each in thermal equilibrium with a third system, are in equilibrium with each other") is one of the fundamental assumptions of thermodynamics; because of Bueche's emphasis on microscopic ideas, he is unable to give the zeroth law much space.

For Objective 2, read Sections 16.1 through 16.3, General Comment 2, study Illustration 16.1, and work Problems B and H. Then read Section 16.14, study Illustration 16.8, and work Problems C and I for Objective 3. Bueche uses the symbol λ for heat conductivity, but we shall use k. For Objective 4, read Sections 16.10 and 16.12, study Illustrations 16.3 through 16.5, and work Problems D and J.

Objective 5 is the most important and comprehensive objective in this module. Read Sections 16.5 and 17.1 through 17.4. Then read General Comments 3 to 9. Study Illustration 17.1 and work Problem 1 in Chapter 17. Finally work Problems E, F, K, and L.

In Section 17.1 Bueche states the first law as $\Delta U = \not{Q} - \Delta W$. If you find the combined presence of differentials (infinitesimal changes) and deltas (finite changes) confusing, you may prefer to remember this law in the form

$\Delta U = Q - W$ (finite changes) or $dU = \not{Q} - \not{W}$ (infinitesimal changes).

(The meaning of the slashes through the differentials for Q and W is explained in General Comment 7.)

After you have completed the reading and problems, do the Practice Test; if you are successful, take a Mastery Test. If you need more help, reread the appropriate sections of the text and General Comments, then work the relevant Additional Problems listed in the Table.

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*Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition.

STUDY GUIDE: Thermodynamics - First Law 3(HR 1)

TEXT: David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974)

SUGGESTED STUDY PROCEDURE

We recommend that you proceed objective by objective, as follows: For Objective 1, read and study Chapter 18, Sections 18-1 to 18-6, and General Comment 1; then study Problem A and work through Problem G.

The thermometer shown in Figure 18-1 may be difficult to understand. The more commonly seen device shown in Figure 1 will also function as a constant-volume gas thermometer; however, it will not give a very accurate reading of the Kelvin temperature (because gases are generally stored at quite high pressure in such cylinders).

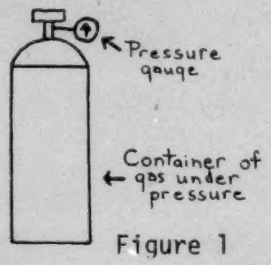


Figure 1

Read and study Chapter 19, Sections 19-1 and 19-2 and Example 1 for Objective 2. Then work Problem 5 in the text and Problems B and H in this study guide.

HALLIDAY AND RESNICK

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text (Chap. 19)	Study Guide	Text (Chap. 19)	
1	Secs. 18-1 to 18-6, General Comment 1	A		G		Chap. 18, Probs. 5, 6
2	Secs. 19-1, 19-2, General Comment 2	B	Ex.[a] 1	H	5	Chap. 19, Probs. 1(a), 23
3	Sec. 19-3	C	Ex. 2	I	11	Chap. 19, Probs. 9, 15
4	Sec. 19-7	D		J	13	Chap. 19, Probs. 3, 4
5	Secs. 19-4 to 19-7, General Comments 3 to 9	E, F	Ex. 3	K, L	27	Chap. 19, Probs. 24, 25, 29 (not easy), 31

[a]Ex. = Example(s).

Your text uses the term "specific heat" to refer to the quantity $c = \Delta Q/(m\,\Delta T)$; we shall employ "specific heat capacity." Read General Comment 2. Then, for Objective 3, read Section 19-3 carefully; study Example 2 of Chapter 19, and work Problem 11. Finally, work Problems C and I.

Read the third paragraph of Section 19-7 (on pp. 367, 368, starting with "Let us consider the boiling process..."). The result you will need to remember for Objective 4 is that for boiling, $Q = mL$, where Q represents the total heat absorbed, m represents the mass of material vaporized, and L symbolizes the heat of vaporization. This formula can also be used for all phase changes (vaporization, condensation, melting, freezing, sublimation), and L is then referred to as the heat of transformation or the latent heat. If you have difficulty with Problems 13, D, and J, read Chapter 16, Section 16-6, "Change of Phase," in University Physics,* and work Problems 16-23 and 16-33.

For Objective 5, read and study Sections 19-4 through 19-7, plus General Comments 3 to 9. Look over Example 3 carefully, and work Problems 27 in Chapter 19 and E, F, K, and L in this study guide. Note: In Figure 19-4 the force $\vec{F}$ is drawn incorrectly. It should be directed upward, since it is the force exerted on the piston by the gas inside the cylinder. With this correction, the work done by the gas is correctly given by $d\vec{W} = \vec{F} \cdot d\vec{s}$, as stated in the text.

When you feel that you have mastered all of the objectives, try the Practice Test. If you are successful, take the Mastery Test; if not, study the appropriate sections, examples, and problems once more, and, if necessary, work some of the Additional Problems listed in the Table.

*Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition.

STUDY GUIDE: Thermodynamics - First Law 3(SZ 1)

TEXT: Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition

SUGGESTED STUDY PROCEDURE

The concepts of this module are developed extremely well in your text. Therefore, we recommend that you read the relevant chapters twice; the first time, read Chapters 15, 16, 17, and 19, Sections 19-1 through 19-11 fairly quickly to get an overview. The second time, proceed objective by objective, working the relevant problems (see Table below) as you go.

For Objective 1, read and study Chapter 15, Sections 15-1 to 15-4, General Comment 1, and work Problems A and G in this study guide. Then study Chapter 16, Sections 16-1 through 16-3 and General Comment 2. Work Problems B and H and Problem 16-3 in your text. Study Chapter 17, Section 17-1 for Objective 3, then do Problems 16-1 in the text and C and I in this study guide. Problem 17-3 in the text is similar to C and I but somewhat more challenging. For Objective 4, read Section 16-6 in Chapter 16 and work Problems D and J. To master Objective 5 (First Law), read Sections 19-1 through 19-8, 19-10, and 19-11, plus General Comments 3 to 9. This is the most comprehensive and important objective; you should think through a response to Problem 19-1, and check your answer; then work out Problems E, F, and K, and study solutions provided carefully. Finally, do Problem L. When you feel that you have mastered all of the objectives, try the Practice Test. If you succeed, take a Mastery Test; if not, study the appropriate sections, examples, and problems, and, if necessary, do some of the Additional Problems listed in the table.

SEARS AND ZEMANSKY

Objective Number	Readings	Problems with Solutions	Assigned Problems		Additional Problems
		Study Guide	Study Guide	Text	
1	Secs. 15-1 to 15-4, General Comment 1	A	G		15-1, 15-2
2	Secs. 16-1 to 16-3, General Comment 2	B	H	16-13	16-5, 16-6, 16-8, 16-10, 16-16
3	Sec. 17-1	C	I	16-1, 17-3	17-5, 17-7 (not easy)
4	Sec. 16-6	D	J		16-23, 16-25, 16-27, 16-33, 16-35
5	Secs. 19-1 to 19-8, 19-10, 19-11, General Comments 3 to 9	E, F	K, L	19-1(c)	19-3, 19-7, 19-9, 19-13 (not easy)

TEXT: Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 1

SUGGESTED STUDY PROCEDURE

In contrast with the strictly macroscopic point of view adopted in this module, Weidner and Sells explain classical thermodynamics largely within the framework of kinetic theory. To get an overview, you should quickly read through, but not study, the following sections: Chapter 18, Sections 18-1 to 18-3 and 18-6 to 18-8; Chapter 19, Section 19-4; Chapter 20, Sections 20-1 to 20-3, 20-5 and 20-6. As you read, keep the macroscopic-microscopic distinction in mind; you will not be expected to learn any aspects of kinetic theory in this module, but the overview will be helpful. For further reading, an excellent treatment of macroscopic thermodynamics is in Chapter 16 of University Physics.*

For your second reading, we recommend that you proceed objective by objective, studying the problems with solutions and working the assigned problems listed in the table. For Objective 1, read Sections 18-1 through 18-3 and General Comment 1, and work Problems A and G in this study guide. Then read the last three paragraphs of Section 20-2 [Eq. (20-1)], Section 20-3 and General Comment 2. Study Example 20-1, and work Problems B and H in this study guide for

WEIDNER AND SELLS

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text	Study Guide	Text	
1	Secs. 18-1 to 18-3, General Comment 1	A		G		18-2
2	Secs. 20-2, 20-3, General Comment 2	B	Ex.[a] 20-1	H		20-3, 20-6
3	Secs. 20-3, 20-6	C	Ex. 20-4	I		20-15, 20-19
4	Sec. 20-5	D	Ex. 20-3(a)	J	20-9	20-10, 20-13, 20-16
5	Secs. 18-6 to 18-8 General Comments 3 to 9	E, F	Ex. 20-2, 20-3(b)	K, L	20-1	20-4, 20-5, 20-8

[a]Ex. = Example(s).

*Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition.

Objective 2. Your text's definition of specific heat capacity ($c = \Delta U/m\,\Delta T$) differs from the definition we shall use below in solving problems ($c = \Delta Q/m\,\Delta T$), but for liquids and solids the two definitions differ by a negligible amount.

For Objective 3, read Section 20-6, study Example 20-4, and work Problems C and I. Read and study Section 20-5 up to the last paragraph on p. 417 for Objective 4; the rest of this section and Figures 20-3 and 20-4 should be skimmed. Work through part (a) of Example 20-3, and do Problem 20-9. Study Problem D and work Problem J of this study guide.

For the First law, Objective 5, read Sections 18-6 through 18-8, 19-4, 20-2, and General Comments 3 to 9. As you read, study Example 20-2 and part (b) of Example 20-3, then work Problem 20-1 and check your answer at the back of the text. Finally work Problems E, F, K, and L.

Note: Weidner and Sells state the first law of thermodynamics as follows: $dQ = dU + dW$ (Section 19-4, p. 386). Later they state the alternative form $\Delta Q = \Delta U + \Delta W$ (Section 20-2, p. 411). We shall use slightly different but equivalent statements in presenting solutions to the problem set: $\Delta U = Q - W$ and $dU = \not{d}Q - \not{d}W$. The sign conventions for U, Q, and W are the same. We also prefer to state the first law with the internal energy, U, on the left and with the heat, Q, and work, W, on the right. The meaning of the slashes through the differentials $\not{d}Q$ and $\not{d}W$ will be explained in General Comment 7.

When you feel prepared, do the Practice Test; if you are successful, take a Mastery Test. If you need more help, reread the appropriate sections of the text or the General Comments and work the relevant Additional Problems listed in the Table.

GENERAL COMMENTS

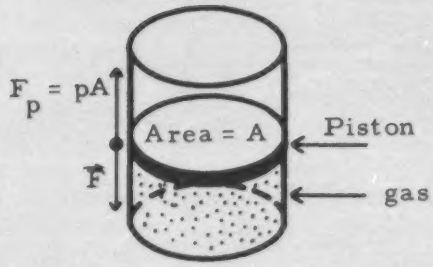
1. Pressure

The pressure p is defined to be the f force per unit area. If some gas is enclosed in a cylinder with a leak-proof (though frictionless) piston having an area A (see Fig. 2), this gas will exert an upward force of magnitude

$$F_p = pA$$

on the piston. An oppositely directed force of equal magnitude will be needed to hold the piston in place. In the diagram, this force is represented by $\vec{F}$.

Figure 2



2. Heat Units

Your text uses units of calories (or kilocalories) for heat, but we shall consistently employ the joule unit (J), in accordance with the International System:

$$1\ J \approx \frac{1}{4.2}\ cal = \frac{1}{4.2 \times 10^3}\ kcal.$$

The remaining comments concern conservation of energy, as stated in the first law of thermodynamics, which is the most important and comprehensive concept of this module.

3. Statement of the FIRST LAW OF THERMODYNAMICS

> For a given system, which undergoes any process beginning and ending in equilibrium, the heat transferred to the system (Q) minus the work done by the system (W) is exactly equal to the change in the internal energy function U. In symbols, $\Delta U = Q - W$

It is important to be clear about the system under consideration and to define the boundaries of the system carefully. For example, if the system is a gas within a cylinder, the cylinder walls and the piston head form the boundaries. The size of the system or the shape of the boundary can change, as when the piston moves, but in order for the first law to be valid one must keep track of the heat and work crossing the boundary. For example, if the piston moves out, then the system does work W on the environment, and if a flame is put under the cylinder then heat is transferred to the system.

We shall only consider systems in which no material enters or leaves, or which can be idealized as such. It is possible to add a term to the first law to take into account matter entering or leaving the system, but this is taken up in more advanced courses. Thermodynamics was originally the study of "heat engines"; that is, systems that absorbed heat and performed work. Therefore, heat was naturally considered positive when it entered a system, and work was positive when it left the system. The same conventions are still used today, and this is the reason for the difference in sign between Q and W in the first law.

4. Definition of Work

From your previous study of mechanics (Work and Energy) you may recall that work on a particle is defined as

$$W = \int \vec{F} \cdot d\vec{r},$$

where the integral is carried out along the path taken by the particle. We use the same definition for the work done by a system in thermodynamics. In cases where a system expands or contracts by volume dV, we can extend this definition to read

$$W = \int_{V_i}^{V_f} p \, dV,$$

where p represents the pressure within the system, and the integral is carried out from the initial volume V_i to the final volume V_f. Notice that work is positive when the system expands, but this tends to decrease the internal energy function because of the minus sign in the first law. It is also useful to realize that $\int p \, dV$ can be represented as the area under the graph of p versus V, and this can sometimes be the best way to calculate W for a given process (see Problems K and L). In addition to changes in the volume of the system, it is important to remember that other types of work can also change the internal energy of a system. For example, electrical and frictional forces can also do work, and this must be included in calculating ΔU.

Work done on a system has the result of increasing the internal energy function. Thus rubbing your hands together rapidly is associated with a rise in temperature of your palms; compressing gas in a cylinder also results in a temperature rise, and rapid stirring of a water-ice mixture brings about melting of some of the ice.

5. Definition of Heat

There is also a way to increase the internal energy of a system without the performance of work. This method is known as heat transfer or heat flow, and it is associated with a difference in temperature across the boundary separating the system and the environment. For example, one could hold one's hands near a fire; a cylinder of gas could be placed in boiling water; and a water-ice mixture could be held above a house radiator. In summary, heat flow is a transfer of energy that brings about changes in system properties; these changes are exactly the same as would be produced by the performance of work, and they are associated with a difference of temperature across the system boundaries.

In this module you study three topics related to heat transfer: conduction, specific heat capacity, and change of phase. These are basically just ways of accounting for or measuring heat transfer Q across the boundary of a system; thus, they provide a way to put numbers into the first law. For example, for a pot of water on the stove, we can calculate Q in three different ways: (1) The temperature difference ΔT between the inside and outside of the bottom, plus its thermal conductivity k, thickness L, and area A lead to a value for $dQ/dt = -kA(\Delta T/L)$. (2) If the water temperature is observed to rise by ΔT, then Q can be found from $Q = mc\,\Delta T$, where m represents the mass of water in the pot, and c represents the known specific heat capacity of water. (3) If the water is allowed to boil and M kg boils away, then, using L to represent the known heat of vaporization of water, $Q = ML$.

6. Equivalence between Internal and Mechanical (Kinetic and Potential) Energy

You should recall the law of conservation of mechanical energy for conservative forces (Conservation of Energy module), which can be stated as

$$K + U(x, y, z) = E,$$

where K (kinetic energy) = $(1/2)mv^2$, U represents the potential energy function, and E represents the total mechanical energy, which is constant. In particular, for motion of a particle (mass m) near the surface of the earth, $U = mgy$, assuming that no forces other than gravity are acting. Mechanical energy can be converted into internal energy in various ways, always through the medium of work. If the forces acting are known, then one can calculate the work and the energy transferred from mechanical to internal form. However, it is usually easier to calculate the energy transfer directly, as in the example below.

Example

A block of ice at 0° C with mass 50 kg, slides along a horizontal surface, starting at a speed of 5.4 m/s and finally coming to rest after traveling 28.3 m. Given the coefficient of sliding friction μ, calculate the heat generated and, assuming that all of the heat is transferred to the ice, the mass of ice melted.

Solution

Rather than calculating the work using the weight, coefficient of friction, and distance traveled, we note that all of the initial kinetic energy is converted into internal energy of the ice. $K = (1/2)m_1v^2$, where $m_1 = 50$ kg, and $v = 5.4$ m/s. Thus

$$K = (1/2)(50 \text{ kg})(5.4 \text{ m/s})^2 = 730 \text{ J}.$$

Now, $\Delta U_{ice} = m_2L$, where m_2 represents the mass of ice melted, and $L = 3.3 \times 10^5$ J/kg, the heat of fusion for water. We set $\Delta U_{ice} = K$, or $m_2L = 730$ J,

$$m_2 = \frac{730 \text{ J}}{3.3 \times 10^5 \text{ J/kg}} = 2.10 \times 10^{-3} \text{ kg}.$$

This is a very small amount of ice melted: In general the amounts of mechanical energy with which we have direct experience (10^2 to 10^3 J) usually produce rather small thermal effects.

7. Differential Form of First Law and Inexact Differentials

The first law can also be written for infinitesimal changes as $dU = \not{d}Q - \not{d}W$. The slashes through the differentials for Q and W indicate that these are not "exact differentials." This is to guard against any misconception that there might exist functions of the thermodynamic variables p, V, T, etc., of which $\not{d}Q$ and $\not{d}W$ would be differentials - there are no such functions. On the other hand, dU is an exact differential, and there does exist a unique function U that depends only on the state of the system and not on the path by means of which the system is brought to that state. There are other such functions of state, for example, p, V, T, and the entropy S, which will be pursued further in the module Second Law and Entropy.

8. Thermodynamic Processes

You should be familiar with two terms used to describe particular types of processes:

Adiabatic refers to processes in which Q = 0. This is usually the result of insulation surrounding the system of interest, but many processes are carried out quickly enough so that appreciable amounts of heat are not transferred to the working substance, and these processes are also referred to as adiabatic. An example of the latter type is the expansion of the burning gases in an automobile engine cylinder. For adiabatic processes, $\Delta U = -W$.

An isobaric process is carried out at constant pressure. All processes carried out with systems open to the atmosphere are isobaric. For an isobaric process, $W = \int p\, dV = p \int dV = p(\Delta V)$; thus, if a change of volume occurs as part of an isobaric process, the work is easy to calculate.

9. Pressure-Volume Graphs

One of the most useful tools in the study of a thermodynamic system is a pressure-volume (pV) graph. In addition to compactness and clarity, pV graphs have the helpful feature that work can be represented as an area under the curve.

Example

Draw a pV graph for two sequential processes: (1) Isobaric expansion of a gas in a cylinder from V_0 to $2V_0$ at p_0, and (2) heating of the cylinder and gas with the piston fixed; the pressure increases from p_0 to $2p_0$, and the volume remains constant at $2V_0$.

Solution

The pV graph is shown in Figure 3. Now calculate the work done, W_1 and W_2, during each process:

$$W_1 = \int p\, dV = p\, \Delta V = p_0(2V_0 - V_0) = p_0V_0.$$

This is represented by the shaded area in the graph.

$$W_2 = \int p\, dV, \quad dV = 0,$$

therefore $W_2 = 0$.

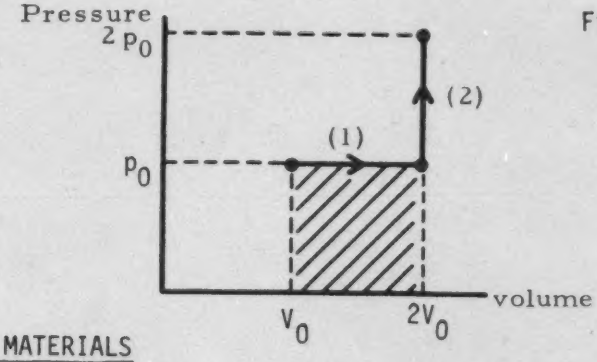


Figure 3

ADDITIONAL LEARNING MATERIALS

S. Angrist and L. Hepler, Order and Chaos (Basic Books, New York, 1967). This is an interesting and easy-to-read book dealing with the basic content and history of thermodynamics. Many fascinating details are given concerning the experiences and personalities of the early thermodynamicists.

Richard Merrill et al. (Eds.), Energy Primer, Solar, Water, Wind, and Biofuels (Portola Institute, 558 Santa Cruz Ave., Menlo Park, Calif., 1974). This contains the meaty article, "Solar Radiation and Its Uses on Earth," with an appendix on "Methods of Estimating Solar Heater Performance," both by John I. Yellott. This article is probably the best introduction to the field; there are numerous opportunities to employ the concepts covered in this module in making estimates of the feasibility and required dimensions of the systems described. This field - utilization of solar energy - is steadily becoming more exciting, and there is a challenging opportunity here for people with knowledge of thermodynamics to make an important social contribution.

Morton Mott-Smith, The Concept of Energy Simply Explained (Dover, New York, 1934). This is slightly more technical than Angrist and Hepler; it also is historical but gives more attention to the kinds of practical engineering problems that James Watt and other early workers were trying to solve. One gets an excellent view of why and how the theory of thermodynamics grew up. For example, the first pV graphs were "indicator cards" drawn by the moving pistons of Watt's engines.

H. C. Van Ness, Understanding Thermodynamics (McGraw-Hill, New York, 1969). This is a slim, enjoyable, and insightful book. It attempts to show, in a conversational way, the plausibility and usefulness of the basic concepts. Topics covered are the first law, reversibility, heat engines, power plants, the second law, and statistical mechanics.

Mark W. Zemansky, Heat and Thermodynamics (McGraw-Hill, New York, 1968), fifth edition. This is an excellent textbook, written at a somewhat more advanced level than this module.

PROBLEM SET WITH SOLUTIONS

A(1). (a) State the zeroth law of thermodynamics and define "thermal equilibrium" and "system."

(b) Suppose that you wish to identify an unknown liquid by measuring its boiling point accurately. You first use a constant-volume gas thermometer to measure the pressure (p) of the confined gas to be 2.6×10^4 Pa* at the triple point of water. Then you bring the same confined gas to equilibrium with the unknown boiling liquid and measure $p = 3.5 \times 10^4$ Pa. What is the temperature of vaporization on the Kelvin scale?

(c) What is the temperature of vaporization on the Celsius scale?

(d) What is the unknown liquid?

Solution

(a) The zeroth law states that two systems in thermal equilibrium with a third system are in thermal equilibrium with each other. Thermal equilibrium can be defined as the state in which two systems are in thermal contact (for example, separated by a thin sheet of copper) and in which changes in the systems have ceased. A system is a portion of matter that we separate (in our minds) from

*The pascal is the SI unit for pressure: 1 Pa = 1 N/m^2.

the environment external to it; the properties of the system itself are described by variables (for example, temperature, pressure, volume, mass, internal energy), and the interaction with the environment is described in terms of quantities transmitted across the system boundaries (e. g., heat and work).

(b) Using the definition $t \propto P$, and recalling that the temperature of the triple point of water is defined to be 273.16 K,

$$\frac{T(K)}{273.16\ K} = \frac{35\ Pa}{26\ Pa}; \qquad T = \frac{35}{26}(273.16) = 368\ K \approx 370\ K.$$

(c) $T(°C) = T(K) - 273.15° = 95°C$.

(d) Since water has a boiling temperature at standard pressure of 100°C, the liquid is probably water. (If the atmospheric pressure were below standard pressure during the experiment, it would explain the observed reduction of the boiling temperature.)

B(2). An aluminum block ($c = 9.1 \times 10^2$ J/kg K) of mass 0.80 kg at a temperature of 275°C is dropped into an aluminum calorimeter cup of mass 0.200 kg containing 1.00 kg of water ($c = 4.2 \times 10^3$ J/kg K) at 20°C. The system is insulated and attains equilibrium at a final temperature T_f.

(a) Using the definition of specific heat capacity, set up the $Q_{total} = 0$ equation.

(b) Solve for T_f.

Solution

(a) $Q_{\text{"gained" by aluminum}} = c_A M_A T_A = c_A M_A (T_f - T_A)$,

$Q_{\text{gained by cup}} = c_c M_c (T_f - T_c)$,

$Q_{\text{gained by water}} = c_w M_w (T_f - T_w)$.

If $Q_{total} = 0$, then $c_A M_A (T_f - T_A) + c_c M_c (T_f - T_c) + c_w M_w (T_f - T_w) = 0$.

(b) $T_f(c_A M_A + c_c M_c + c_w M_w) = c_A M_A T_A + c_c M_c T_c + c_w M_w T_w$,

$$T_f = \frac{c_A M_A T_A + c_c M_c T_c + c_w M_w T_w}{c_A M_A + c_c M_c + c_w M_w} = \frac{2.878 \times 10^5\ J}{5.1 \times 10^3\ J/°C} = 56°C.$$

Note that heat is actually lost by the aluminum, but this is automatically taken into account by the fact that we wrote all temperature changes as T(later) - T(earlier) in part (a), and we put all of the heat terms on one side of the equation. If you set up the equation Q(gained) = Q(lost), as Bueche, Halliday and Resnick, and Weidner and Sells do, then you must be careful to make all ΔT's positive, that is, T(higher) - T(lower).

C(3). A copper rod (length 0.35 m) has a circular cross section with radius r = 0.050 m. The two ends are kept at different, fixed temperatures, and the sides are insulated. The energy transfer along the rod is observed to be 8.4×10^2 W, and the temperature in the middle is 145°C. For copper the thermal conductivity $k = 3.8 \times 10^2$ W/m K [1 watt (W) = 10^3 mW = 1 J/s].

(a) Draw a sketch of the setup.

(b) What are the temperatures of the two ends?

Solution

(a) Note: You are only expected to produce a sketch, similar to Figure 4 with L = 0.35 m and $r = 5.0 \times 10^{-2}$ m.

(b) Since the rod has constant cross section, we can put $dT/dx = (T_2 - T_1)/L$, and the heat equation becomes

$$\frac{dQ}{dt} = -kA\left(\frac{T_2 - T_1}{L}\right) \quad \text{or} \quad (T_2 - T_1) = -\left(\frac{L}{kA}\right)\left(\frac{dQ}{dt}\right),$$

and

$$T_2 - T_1 = \frac{0.35 \text{ m}}{(3.8 \times 10^2 \text{ W/m K})(2.50 \times 10^{-3} \text{ m}^2)}(8.4 \times 10^2 \text{ W}) = 98 \text{ K};$$

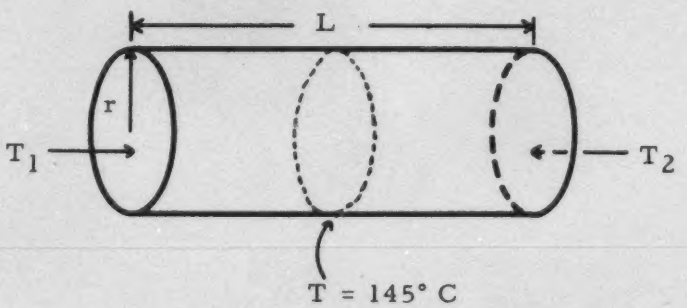
$$T_2 = 145 + 98/2 = 194°C; \qquad T_1 = 145 - 98/2 = 96°C.$$

D(4). A block of ice, mass 0.20 kg and temperature 0°C, is dropped into a brass calorimeter cup of mass 0.100 kg containing 0.300 kg water at 80°C. Assuming that no heat is lost to the environment, find T_f, the final equilibrium temperature.

$c_{brass} = 4.2 \times 10^2$ J/kg K; $\quad L_{ice\ melting} = 3.3 \times 10^5$ J/kg;

$c_{water} = 4.2 \times 10^3$ J/kg K.

Figure 4



(a) Write an expression for Q_1, the heat gained by the ice. Be sure to include both the heat involved involved in melting the ice ($Q = ML$) and in warming the resulting water.

(b) Write an expression for Q_2, the heat lost by the cup and the water.

(c) Equate the expressions from (a) and (b), solve for T_f, and evaluate.

Solution

(a) $Q_1 = M_I L + c_w M_I (T_f - 0)$.

(b) $Q_2 = c_{cup} M_{cup} (80 - T_f) + c_w M_w (80 - T_f)$.

(c) $Q_1 = Q_2$, $\quad c_w M_I T_f + c_{cup} M_{cup} T_f + c_w M_w T_f = c_{cup} M_{cup} 80 + c_w M_w 80 - M_I L$,

$$T_f = \frac{c_c M_c 80 + c_w M_w 80 - M_I L}{c_w M_I + c_c M_c + c_w M_w} = \frac{3.816 \times 10^4 \text{ J}}{2.142 \times 10^3 \text{ J/k}} \approx 18°\text{C}.$$

E(5). (a) State the first law of thermodynamics, and define all the terms appearing in it.

(b) The density of liquid water at 100°C and 1 atm ($\simeq 10^5$ Pa) is 1.00×10^3 kg/m^3, and the heat of vaporization is 2.30×10^6 J/kg. The density of water vapor under the same conditions is 0.58 kg/m^3. Considering 1.00 kg of water as the system calculate the change in volume ΔV during vaporization.

(c) Using $W = \int p \, dV$, calculate the work done <u>by</u> the system <u>on</u> the surrounding atmosphere. Is this work positive or negative?

(d) Set up the first law for this problem; be careful about the signs for Q and W, and calculate ΔU.

Solution

(a) $\Delta U = Q - W$.

(b) $\Delta V = V_{steam} - V_{water} = 1/\rho_s - 1/\rho_w = 1.72 \text{ m}^3$, where ρ_w is the density of water and ρ_s is the density of steam.

(c) $W = \int p \, dV = p \, \Delta V$ (since p is constant), $W = (10^5 \text{ Pa})(1.72 \text{ m}^3) = 1.70 \times 10^5$ J. This is positive work since the system expanded; work was done <u>by</u> the system on the environment.

(d) $U = +2.3 \times 10^6 \text{ J/kg} - (1.70 \times 10^5 \text{ J}) = 2.13 \times 10^6 \text{ J/kg}$.

F(5). (a) A car (M = 1600 kg) is traveling at 56 mph (25.0 m/s) when it brakes to a halt. The car has four iron brake drums (10.0 kg each) with a specific heat capacity $c = 4.6 \times 10^2$ J/kg K. The brakes are air-cooled, and each drum loses 15 000 J to the air during the braking. Assume that the internal energies of the four brake drums increase equally and that the other parts of the car and the road suffer no change in internal energy. What is the change in temperature of the brake drums? (Start from the first law.)

(b) A waterfall is 75 m high. The water has a temperature of 20.0°C above the waterfall and is flowing at 4.0 m/s. Calculate the expected temperature at the bottom. Start from the first law; neglect any transfer of heat to the ground or to the air. Water has a density of 10^3 kg/m^3 and a specific heat of 4.2×10^3 J/kg K.

Solution

(a) $\Delta U = Q - W$. Considering one brake drum as the system, work is done on the drums, and we can calculate this work by equating it to one-fourth of the original kinetic energy of the car; thus

$$W = -(\tfrac{1}{4})(\tfrac{1}{2})mv^2 = -(\tfrac{1}{4})(\tfrac{1}{2})(1600\text{ kg})(25.0\text{ m/s})^2 = -1.25 \times 10^5\text{ J}, \quad Q = -1.50 \times 10^4\text{ J};$$

therefore from the first law

$$\Delta U = (-1.50 \times 10^4\text{ J}) - (-1.25 \times 10^5\text{ J}) = +1.10 \times 10^5\text{ J}.$$

To find the temperature rise we use the definition of specific heat capacity:

$$T = \frac{Q}{cM} = \frac{1.10 \times 10^5\text{ J}}{(4.6 \times 10^2\text{ J/kg})(10.0\text{ kg})} = 24°\text{C}.$$

(b) $\Delta U = Q - W$. The system is a given mass of water. $Q = 0$, since we are neglecting heat transfer to the ground and air. The work is done on the system by the earth and the surrounding water; therefore W is negative. We can calculate W by equating it with the kinetic plus gravitational potential energy of the water at the top of the waterfall,

$$W = (1/2)mv^2 + mgh.$$

To calculate the temperature rise, we use the definition of specific heat,

$$cmT = (\tfrac{1}{2})mv^2 + mgh \quad \text{and} \quad T = \frac{(1/2)v^2 + gh}{c} = 0.18°\text{C}.$$

We expect a temperature of 20.18°C, which would be difficult to measure. (Joule actually tried this experiment, unsuccessfully.) As noted above in General Comment 5, the thermal effect of mechanical work is usually rather small.

Problems

G(1). (a) On a brisk autumn day (5.0°C) you inflate the tires of your car to a pressure of 44 lb/in.^{2} (2.90×10^5 Pa). (Your tire gauge reads 29 lb/in.^{2} or 1.9×10^5 Pa, but you recall that this is the excess above atmospheric pressure, which is about 15 lb/in.^{2} or 1.00×10^5 Pa.) Assuming that the tires and air inside are in equilibrium with the outside air, what is the temperature of the air inside on the Kelvin scale?

(b) After a fast trip down the Interstate, you again measure the pressure and find that the gauge now reads 42 lb/in.^{2} (2.80×10^5 Pa). What is the temperature (in kelvins) now of the air inside the tires? (Assume that the volume of air remains constant.)

H(2). Figure 5 shows a flow calorimeter. A liquid of density 1200 kg/m^3 flows through at the rate of 1.20×10^{-5} m^3/s. The heating element supplies 400 W, and, after steady state is achieved, it is observed that the liquid flowing out is 20.0° warmer than the liquid entering.

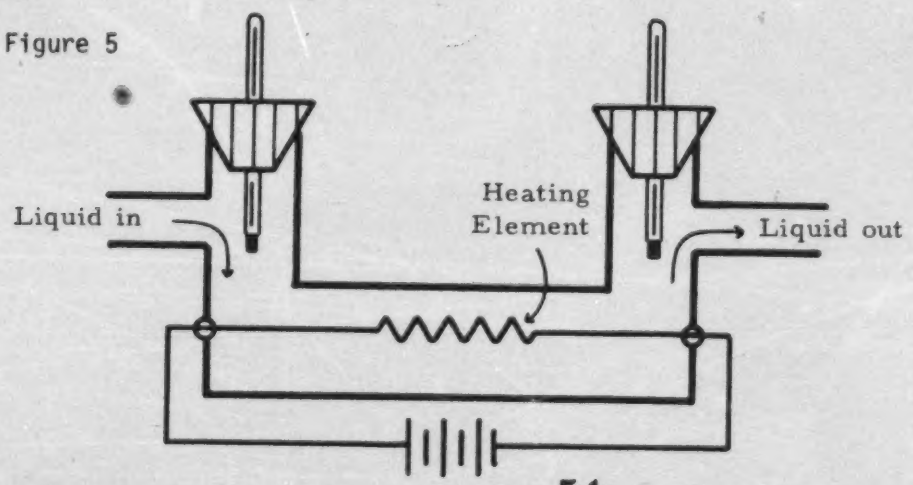
(a) Assume that no heat is lost to the environment, and consider the mass of liquid flowing through the system in t seconds to be the system. Set up an equation for Q_1 (the amount of heat gained by this system in time t) in terms of M (the mass of liquid flowing through in time t), c (the specific heat of the liquid), and T (the temperature change of the liquid).

(b) Find Q_2, the amount of heat lost by the heating element in t seconds.

(c) Using the results of (a) and (b), set up the $Q_1 = Q_2$ equation, solve for the specific heat capacity, and evaluate the resulting expression.

I(3). This problem is illustrative of the way heating systems for buildings are designed. Assume that the building is a cube 10.0 m on a side with walls of wood 0.20 m thick. For wood, $k = 8.4 \times 10^{-2}$ W/m K. If you wish to keep the inside of the cube at 20°C (68°F) when the outside temperature goes down as low as 0°C (32°F), calculate the required capacity (power) of the heating system as follows:

Figure 5



(a) The area A of one wall of the cube is ____ m^2.

(b) Therefore, since the cube has ____ sides, the total surface area A of the building is ____ m^2.

(c) Therefore the cube is equivalent to a large, flat conductor of constant cross-sectional area A, and length (thickness) L = ____ m; the difference in temperature between the two sides is $T_2 - T_1$ = ____ °C.

(d) Therefore, dT/dx is constant throughout the conductor, and we can put

$dT/dx = (T_2 - T_1)/L$ = ____ °C/m.

(e) Using the values above, evaluate the required capacity (power) of the heater, dQ/dt = ____ W.

(f) Estimate whether the magnitude of the answer is reasonable.

J(4). If 0.150 kg of ice and 0.200 kg of water in thermal equilibrium are heated to 60°C by being mixed with M kg of steam at 100°C, how much water will be in the final mixture? Use

$L_{\text{ice melting}} = 3.30 \times 10^5$ J/kg, $L_{\text{water vaporizing}} = 2.30 \times 10^6$ J/kg, and $c_{\text{water}} = 4.2 \times 10^3$ J/kg K.

(a) Write an expression for Q_1, the heat gained by the ice-and-water system. Be sure to include the heat involved in both melting the ice and raising the temperature of the melted ice and water.

(b) Write an expression for Q_2, the heat lost by the steam. Include both the heat to condense the steam and to lower the temperature of the resulting liquid water.

(c) Equate the expressions from (a) and (b), and find the value of M.

K(5). Figure 6 shows an elementary form of steam engine, the Newcomen engine. [You may be interested to know that this engine once represented the vanguard of technology, and it was used for quite some time to pump water out of the mines of England, in spite of its shocking inefficiency (by today's standards). Its success and its defects provided the stimulation for James Watt to invent the modern steam engine.] The engine operates as follows:

Stage 1: Constant volume heating - heat flowing into the boiler changes some of the water to steam, gradually increasing the pressure against the piston. The piston is initially held against the stops by a constant force F, a combination of the weight of the piston and the atmospheric pressure on the top of the piston. During this stage, the volume of the system is constant, and the pressure rises from 0 to p_1 Stage 2: Constant pressure expansion - when the boiler pressure reaches p_1, then the upward

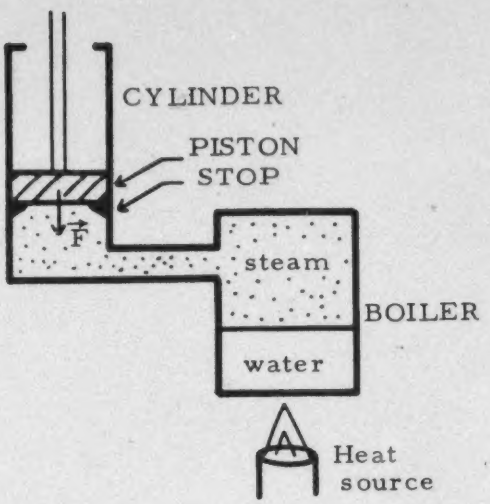


Figure 6

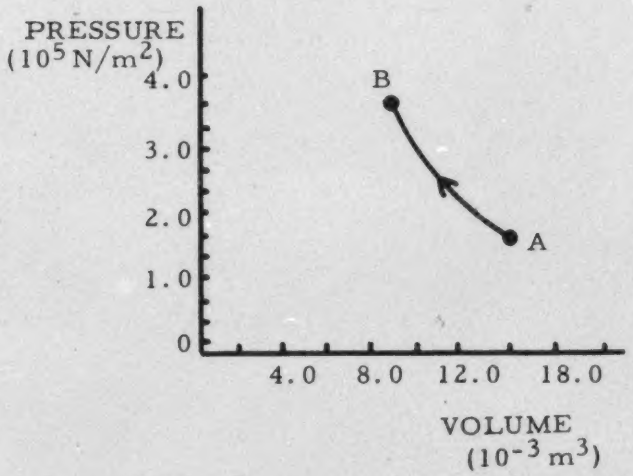
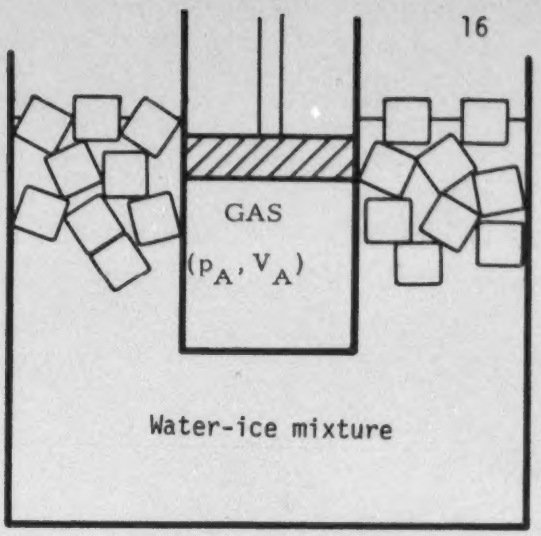
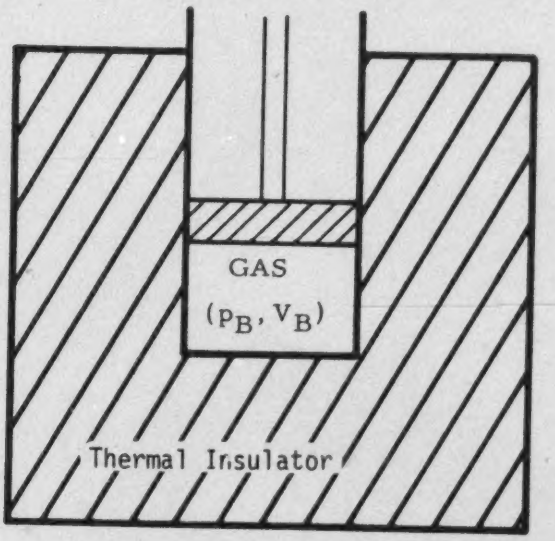
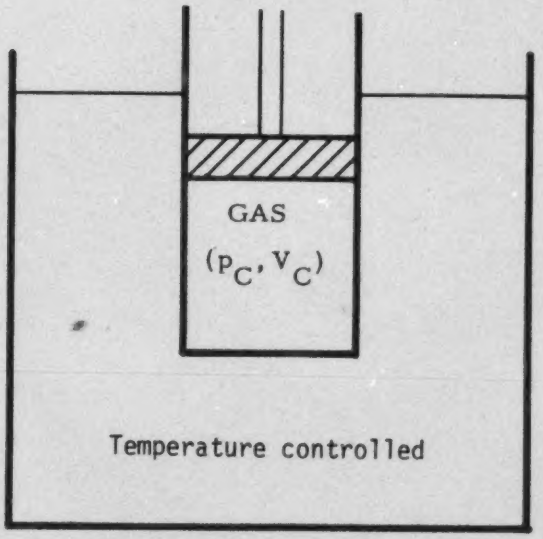


Figure 8

Figure 7







force just balances F, and the piston rises slowly. The pressure of the system remains constant as the volume of the system increases from V_1 to V_2.

During Stage 1, the system gains $Q_1 = 2.10 \times 10^5$ J, and during Stage 2, the system gains $Q_2 = 2.90 \times 10^5$ J; also $p_1 = 4.0 \times 10^5$ Pa, $V_1 = 0.100$ m^3, and $V_2 = 0.200$ m^3.

(a) Draw a pressure-volume graph for Stages 1 and 2.

(b) Which of the following terms do we use to describe Stage 1 and Stage 2: isobaric, adiabatic, or neither?

(c) Give the value of W_1, the work done during Stage 1.

(d) Find the value of ΔU_1, the change in internal energy in Stage 1; use the first law of thermodynamics.

(e) Calculate W_2, the work done during Stage 2. Justify the sign of the answer, and shade the area on the pV graph corresponding to W_2.

(f) Calculate ΔU_2, the change in internal energy during Stage 2. Justify the sign of the answer.

(g) For the complete process, Stage 1 plus Stage 2, give values for W, Q, and ΔU.

L(5). A cylinder contains a sample of gas confined by a piston as in Figure 7. The cylinder is immersed in a water-ice mixture, and the gas is in thermal equilibrium with pressure $p_A = 1.50 \times 10^5$ Pa and volume $V_A = 0.0140$ m^3. We shall refer to this as State A of the system. The gas then undergoes the following sequential changes:

Process 1. The cylinder is removed from the ice-water bath, surrounded with an insulating jacket, and the piston is gradually pushed in. In other words, the gas is compressed adiabatically. The final state of the gas is now State B (see pV graph) with $p_B = 3.6 \times 10^5$ Pa and $V_B = 8.3 \times 10^{-3}$ m^3. During this process the gas has done work W_1.

Process 2. The temperature of the gas is measured, and the cylinder is placed in a controllable bath at the same temperature. The piston is now allowed to move out gradually, and the temperature of the bath (and thus the gas) is controlled in such a way as to keep $p = -kV + c$, with k and c constant (see Figure 7). This process is continued until the piston is at the same position as in State A, and the gas is said to be in State C, $p_C = 2.40 \times 10^5$ Pa and $V_C = V_A$. During Process 2 the gas does work W_2 and gains heat Q_2.

Process 3. The piston is fixed in place; the cylinder is placed back in the ice-water mixture and is allowed to come to thermal equilibrium. During this process the heat flow is Q_3, and the work done is W_3.

(a) Process 1 and States A and B are shown on the accompanying pV graph (Fig. 8). Show Processes 2 and 3 and State C on this graph.

(b) Calculate W_2.

(c) Suppose that during Process 2, the net heat gained from the bath is $Q_2 = +3040$ J. Calculate the change in the internal energy function during Process 2, ΔU_2.

(d) Suppose that during Process 3, 5.7×10^{-3} kg of ice melted in the bath. Using $L = 3.30 \times 10^5$ J/kg, calculate Q_3.

(e) Find the change in internal energy during Process 3, ΔU_3.

(f) Find the change in internal energy during Process 1, ΔU_1. Hint: Use the results of (c) and (e).

(g) Using the result of (f), find W_1.

(h) Combine your results for W_1, W_2, and W_3 to find the total work done during the cycle.

(i) Combine your results for Q_1, Q_2, and Q_3 to find the total heat added during the cycle.

(j) Combine the results for W_1, W_2, and W_3 and Q_1, Q_2, and Q_3 to determine whether, over the entire cycle, work input is being converted to heat output or heat input is converted to work output. How many joules of heat (or work) are converted over the entire cycle?

Solutions

G(1). (a) T = 278 K; (b) T = 364 K. If your answer is 271 K, you forgot that the gauge represents the excess over atmospheric pressure.

H(2). (a) $Q_1 = cMT$. We also must remember that

$$M = (1200 \text{ kg/m}^3)(1.20 \times 10^{-5} \text{ m}^3/\text{s})t = 1.44 \times 10^{-2} t \text{ kg}.$$

(b) $Q_2 = 400t$ J. (c) $Q_1 = Q_2$, $cM\,\Delta T = 400t$,

$$c = 400t/M\,\Delta T = 400t/(1.44 \times 10^{-2} t \text{ kg})(20.0 \text{ K}) = 1.39 \times 10^3 \text{ J/kg K}.$$

I(3). (a) $A_s = 100$ m^2. (b) 6 sides, $A_{total} = 600$ m^2.

(c) L = 0.200 m. (d) $(T_2 - T_1)/L = (20.0°\text{C} - 0.0°\text{C})/0.200 \text{ m} = 100°\text{C/m}$.

(e) $dQ/dt = -kA[(T_2 - T_1)/L] = -(8.4 \times 10^{-2} \text{ W/m K})(600 \text{ m}^2)(100°\text{C/m}) = 5000$ W.

(f) It seems reasonable that fifty 100-W bulbs would keep a 10-m cube comfortable inside when the outside temperature is 0°C (32°F).

J(4). (a) $Q_1 = (0.150)(3.30 \times 10^5) + (4.2 \times 10^3)(0.35)(60) = 1.38 \times 10^5$ J.

$Q_2 = M(2.30 \times 10^6) + (4.2 \times 10^3)M(40) = 2.47 \times 10^6 M$ J.

(c) $M = (1.38 \times 10^5)/(2.47 \times 10^6) = 0.056$ kg. Therefore there will be (0.150 + 0.200 + 0.056) kg = 0.41 kg of water in the final mixture.

K(5). (a) The pressure volume graph is shown in Figure 9.

(b) Stage 1 = neither (isochoric); Stage 2 = isobaric.

(c) $W_1 = 0$ (because $\Delta V = 0$).

(d) $\Delta U_1 = Q_1 - W_1 = Q_1 = 2.10 \times 10^5$ J.

(e) $W_2 = \int p\, dV = p\, \Delta V = (4.0 \times 10^5 \text{ Pa})(0.200 - 0.100)\ m^3 = 4.0 \times 10^4$ J.

This is positive because the system expanded and did work on the environment.

(f) $\Delta U_2 = Q_2 - W_2 = (2.90 \times 10^5 \text{ J}) - (4.0 \times 10^4 \text{ J}) = 2.50 \times 10^5$ J. This is positive; the system gained more energy by heat flow than it lost by doing work.

(g) $W_{total} = W_1 + W_2 = 4.0 \times 10^4$ J. $Q_{total} = Q_1 + Q_2 = 5.0 \times 10^5$ J.

$\Delta U_{total} = \Delta U_1 + \Delta U_2 = 4.6 \times 10^5$ J.

Note that if we proceed by means of a different path, say by means of Processes 3 and 4 on the graph (Fig. 9), we would get different values for W and Q but the same value for ΔU.

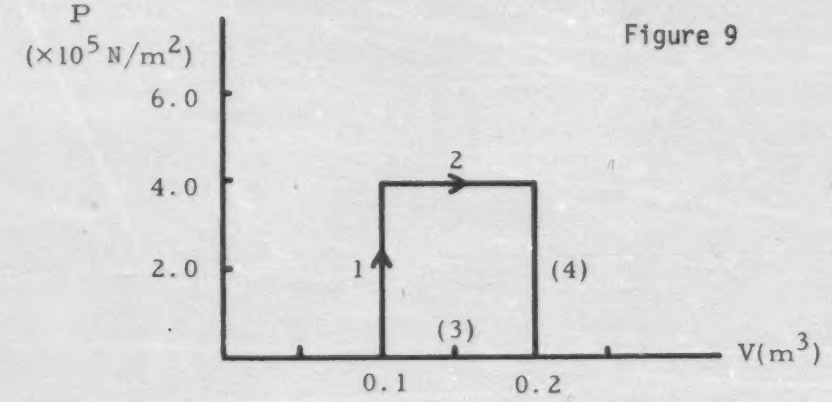
L(5). (a) See Figure 10. (Paths 2 and 3 are represented by straight lines.)

(b) W_2 = area under Path 2 = $P_C(V_B - V_A) + (1/2)(P_B - P_C)(V_C - V_B)$

$= (-1.368 \times 10^3 \text{ J}) + (-3.42 \times 10^2 \text{ J}) = +1.71 \times 10^3$ J.

This is positive, as it should be, since the system expands and does work on the environment.

Figure 9



(c) $\Delta U_2 = Q_2 - W_2 = +3940\ J - (+1710\ J) = +1330\ J.$

(d) $Q_3 = M_{ice\ melted} L = -(5.7 \times 10^{-3}\ kg)(3.30 \times 10^5\ J/kg) = -1880\ J.$

This is negative because heat was lost by the gas.

(e) $\Delta U_3 = Q_3 - W_3$; $W_3 = 0$ since $V = 0$; therefore $\Delta U_3 = Q_3 = -1880\ J.$

(f) $\Delta U_1 = -(\Delta U_2 + \Delta U_3) = -(1330\ J - 1880\ J) = +550\ J.$

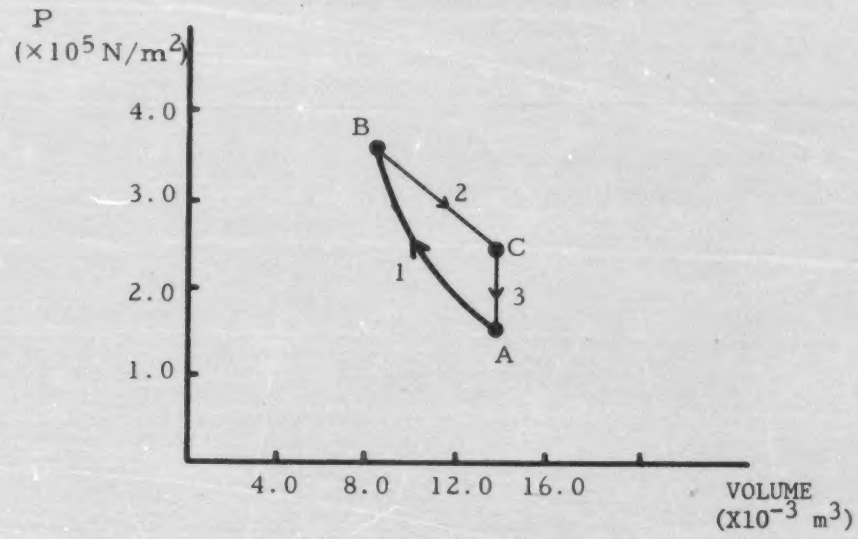
(g) $\Delta U_1 = Q_1 - W_1$, and since Process 1 is adiabatic, $Q_1 = 0$; therefore, $\Delta U_1 = -W_1$, and $W = -550\ J$. This is negative, as it should be, since the system is contracting, and work is being done on the system.

(h) $W_{total} = W_1 + W_2 + W_3 = -550\ J + 1710\ J + 0 = +1160\ J.$

(i) $Q_{total} = Q_1 + Q_2 + Q_3 = 0 + 3040\ J - 1880\ J = +1160\ J.$

(j) Heat input is being converted to work output. One can also determine this by glancing at the pV graph; the area under the curve directed toward increasing V (Process 2) is greater than the area under the curve directed toward decreasing V (Process 1); therefore, the net work is positive.

Figure 10



PRACTICE TEST

1. (a) State the zeroth law of thermodynamics.

   (b) A tank of oxygen gas is at a pressure of 3.8×10^5 Pa at 20.0°C. The tank is dropped into a vat of boiling water (T = 100°C). Calculate the new pressure of the gas.

2. A "solar house" has storage facilities for 4.0×10^9 J. Compare the space requirements for this storage on the assumption that (a) the heat is stored in water ($c = 4.0 \times 10^3$ J/kg K) heated from a minimum temperature of 20.0°C to 40°C; or (b) the heat is stored in Glauber salt heated in the same temperature range. Glauber salt has the following properties (approx): Specific heat capacity (solid): 2.00×10^3 J/kg K. Specific heat capacity (liquid): 3.00×10^3 J/kg K. Density: 1.00×10^3 kg/m^3. Heat of fusion: 2.00×10^5 J/kg. Melting point: 32°C.

3. A rubber ball ($c = 1.20 \times 10^3$ J/kg K) is dropped from a height of 2.00 m onto the floor, where it bounces for a while and finally comes to rest. Find the rise in temperature of the ball, assuming that half of the original energy of the ball went into internal energy of the ball.

4. The pV diagram in Figure 11 represents several processes that may occur in a certain thermodynamic system. In the process 1 → 2 → 3, 90 J of heat enters the system, and 40 J of work is done by the system. In the process 1 → 4 → 3, 10.0 J of work is done by the system. In the direct process 3 → 1, the amount of work is 22.0 J.

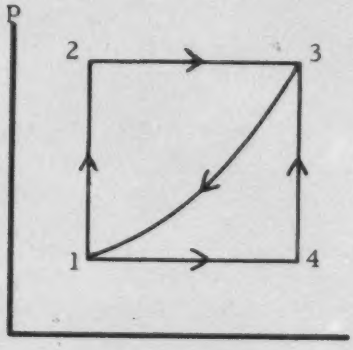
   (a) How much heat enters or leaves the system in the process 1 → 4 → 3?

   (b) How much heat enters or leaves the system during the direct process 3 → 1?

   (c) How much work is done during the cycle 1 → 2 → 3 → 1?

   (d) If $\Delta U_{12} = 30.0$ J, how much work is done during the process 1 → 2, and how much heat enters or leaves the system?

Figure 11



Practice Test Answers

1. (a) If two systems are in thermal equilibrium with a third system, then they are in thermal equilibrium with each other.
   (b) 4.8×10^5 Pa.
2. (a) Volume (water) = 48 m^3. (b) Volume (salt) = 16 m^3.
3. $\Delta T = 1.6 \times 10^{-2}$°C.
4. (a) +60 J.
   (b) −72 J. (W for 3 → 1 is negative.)
   (c) +18.0 J.
   (d) $W_{12} = 0$; $Q_{12} = +30.0$ J.

TEMPERATURE, HEAT, AND THERMODYNAMICS

Date _____

Mastery Test Form A

pass recycle

1 2 3 4 5

Name _____ Tutor _____

1. (a) State the zeroth law of thermodynamics and define the term "system."

   (b) You connect a spherical stainless-steel container to a manometer, which measures the actual pressure of the gas inside the container. When this container is immersed in water at 0°C, the manometer reads a pressure of 2.5×10^5 Pa. But when it is placed in a crucible of molten zinc, the pressure reading quickly rises to 6.0×10^5 Pa. What is the temperature of the zinc in kelvins?

2. A hot 0.300-kg copper object ($c = 3.8 \times 10^2$ J/kg K) is dropped into a perfectly insulated container containing 0.200 kg of water at 20.0°C. This causes the water to boil, 0.0050 kg being converted to steam. The heat of vaporization for water is 2.30×10^6 J/kg. Calculate the original temperature of the copper object.

3. A cylinder containing a sample of gas is immersed in a mixture of water and ice. The initial state of the gas is A, with pressure, volume, and temperature P_0, V_0, and T_0. The piston is very slowly pushed in until the volume is $V_0/4$ and the pressure has increased. We shall call this State B. Then the piston is very rapidly (i.e., adiabatically) pulled out again to its original position, bringing the gas to State C. [$P_C = (1/2)P_0$ and $T_C = 2T_0$.] The gas is then allowed to interact with the ice-water mixture without further changes, and it returns to State A after a period of time.

   (a) Show the three states and processes on a pV diagram, and justify your drawing qualitatively.

   (b) Make a table showing the sign (+, -, or 0) of the heat transferred and the work done by the gas during each process.

   (c) Has the device converted heat to work, or work to heat? If 2.00×10^4 J has been converted, calculate the mass of water frozen or ice melted. The heat of fusion of water is approximately 3.30×10^5 J/kg.

TEMPERATURE, HEAT, AND THERMODYNAMICS Date _____

Mastery Test Form B

pass recycle

1 2 3 4 5

Name _____ Tutor _____

1. You blow up and tie off a balloon from a bottle of gas at 2.00×10^5 Pa pressure and 25.0°C. Assume that the balloon comes to equilibrium at this temperature and pressure. Then you heat the balloon until it bursts at 60°C. What is the bursting pressure?

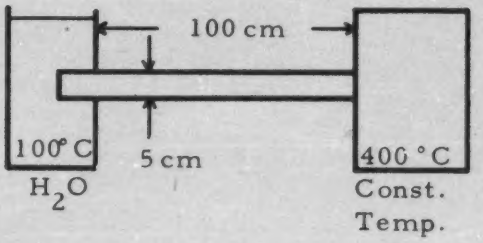
2. The copper bar of square cross section (s = 0.050 m) in Figure 1 with L = 1.00 m has one end immersed in an insulated water bath at 100°C and at atmospheric pressure and the other end connected to a constant-temperature reservoir at 400°C. How long does it take to vaporize 0.300 kg of the water? The thermal conductivity for copper is 4.5×10^2 W/m K, and the heat of vaporization of water is 2.30×10^6 J/kg.

3. (a) State the first law of thermodynamics.

   (b) Define the following in one or two sentences: system, equilibrium, work, heat, internal energy.

   (c) When water freezes at atmospheric pressure, its volume increases by about 10%. Is ΔU for this process larger or smaller in magnitude than the heat of fusion 3.30×10^5 J/kg)? By how much? You may be asked to justify your answer in terms of the first law. (1 atm = 1.00×10^5 Pa; $\rho_{water} = 10^3$ kg/m^3.)

Figure 1



TEMPERATURE, HEAT, AND THERMODYNAMICS

Date _____

pass recycle

Mastery Test Form C

1 2 3 4 5

Name _____ Tutor _____

1. You bring the probe of a constant-volume gas thermometer into equilibrium with a boiling water bath at 92°C. You measure the pressure to be 1.60 $\times 10^5$ Pa. Then you bring the probe to equilibrium with a liquid-solid mixture of an unknown substance. The pressure is then measured to be 0.60 $\times 10^5$ Pa. What is the melting point (in kelvins) of the unknown substance?

2. A copper kettle with a base area of 10^{-2} m^2 evaporates water at a rate of 4.6×10^{-3} kg/s. What is the temperature gradient $\Delta T/\Delta x$ across the bottom of the kettle? ($L_{vap} = 2.30 \times 10^6$ J/kg; $k_{copper} = 4.5 \times 10^2$ W/m K.)

3. Grapes are placed in a juice press at Point 1 in Figure 1. As the top is slowly depressed, the volume decreases and the average pressure exerted on the sides of the juice press increases, until Point 2 is reached. The grapes burst, and the juice runs at approximately constant pressure until Point 3. Since only skins and seeds are left, further increase in pressure (to Point 4) makes no significant change in volume. $Q_{12} = -10.0$ J of heat flows out during process 1 → 2; there is no heat flow during process 2 → 3, but juice flows out, carrying away an internal energy $\Delta U_j = 200$ J; there is no heat flow in process 3 → 4; $V_1 = 3.00 \times 10^{-2}$ m^3, $V_2 = 2.00 \times 10^{-2}$ m^3, $V_3 = 1.00 \times 10^{-2}$ m^3, and $p_2 = 4.0 \times 10^4$ Pa. (Note: Pressure here is taken relative to atmospheric pressure present.)

   (a) How much work is done in the complete process 1 → 2 → 3 → 4?

   (b) What is the change in internal energy of the grapes in the juice press, $U_4 - U_1$, during the complete process?

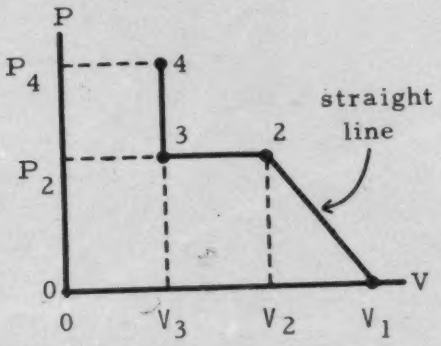


Figure 1

MASTERY TEST GRADING KEY - Form A

1. What To Look For: The zeroth-law statement should be fairly close to the one given below. There are various possibilities for the definition of system.

   Solution: (a) Two systems in thermal equilibrium with a third are in thermal equilibrium with each other. A system is a portion of matter that we separate in our minds from the environment external to it.

   (b) $T = (P_T/P_{273\ K})(273\ K) = [(6.0 \times 10^5)/(2.50 \times 10^5)](273) = 655\ K.$

2. What To Look For: The student should start with Q = 0 or $Q_{lost} = Q_{gained}$.

   Solution: $Q_{lost} = Q_{gained}$, $\quad c_c M_c \Delta T_c = c_w M_w \Delta T_w + M_w L_{vap}$,

$$T_{final} = \frac{c_w M_w \Delta T_w - M L_{vap}}{c_c M_{vap}} + 100°C$$

$$= \frac{(4.2 \times 10^3\ J/kg\ K)(0.200\ kg)(80\ K) - (0.0050\ kg)(2.30 \times 10^6\ J/kg)}{(3.8 \times 10^2\ J/kg\ K)(0.300\ kg)}$$

$$+ 100°C = 790°C.$$

3. What To Look For: (a) The diagram should be a "triangle" with B higher than A and C and with AC a vertical straight line. The students should not be penalized if they draw straight lines or different types of curves for AB and BC. (c) Answer should include 3 parts: work to heat, ice melted, and the amount.

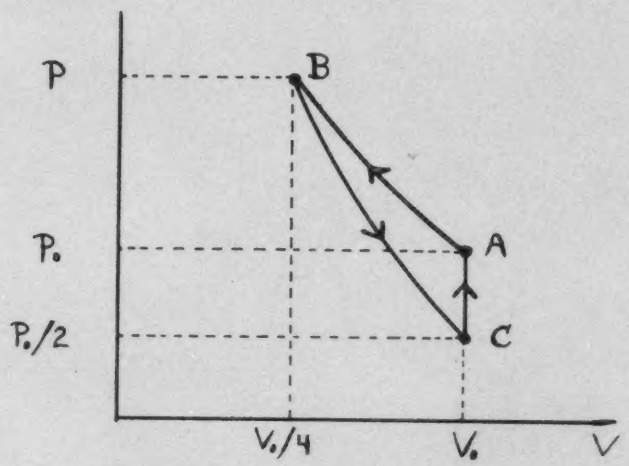


Figure 14

Solution: (a) See Figure 14. The process A → B is a compression; therefore the line must go to the left, and up because the problem states that the pressure rises. The line for B → C must go down and to the right because the problem states that the pressure $p_C = (1/2)p_0$, and it is an expansion. Line CA must be vertical because the piston stays fixed.

(b)

	AB	BC	CA
heat	-	0	-
work	-	+	0

(c) Since the area under AB is negative and greater than the area under BC, which is positive, we expect that work is converted to heat.

$$M(\text{melted}) = \frac{2.00 \times 10^4 \text{ J}}{3.30 \times 10^5 \text{ J/kg}} = 6.1 \times 10^{-2} \text{ kg}.$$

MASTERY TEST GRADING KEY - Form B

1. What To Look For: Student should convert temperatures first to Kelvin scale. If not, mark incorrect and discuss definition of temperature in terms of pressure. Show how to convert from degrees Celsius to kelvins.

   Solution: $p_{burst} = \frac{p_{25.0°C} T_{burst}}{298\ K} = \frac{(2.00 \times 10^5\ Pa)(333\ K)}{298\ K} = 2.23 \times 10^5\ Pa.$

2. What To Look For: Student should use both conduction formula and latent-heat formula.

   Solution: $dQ/dt = Q/t$ because of steady state.

   $dt/dx = \Delta T/\ell$ because of uniform cross section.

   $Q/t = -kA(\Delta T/\ell)$; $Q = ML$,

   $$t = \frac{ML\ell}{kA\,\Delta T} = \frac{(0.300\ kg)(2.30 \times 10^6\ J/kg)(1.00\ m)}{(4.5 \times 10^2\ W/m\ K)(0.050\ m)^2(300\ K)} = 2.0 \times 10^3\ s.$$

3. What To Look For: (a) Student statements do not need to be this comprehensive, but the underlined ideas should be stated explicitly, or at least alluded to. (c) Student should calculate work. Sign is important, as well as number. . . . . Student could say that ΔU is more negative than Q, therefore $\Delta U < Q$, but make sure this is properly explained and that Q and ΔU have the proper signs.

   Solution: (a) For a system that undergoes any process beginning and ending in equilibrium, the heat transferred to the system (Q) minus the work done by the system (W) is exactly equal to the change in the internal energy function U. $\Delta U = Q - W$.

   (b) A system is a portion of matter that we isolate, in our minds, from the environment. A system is in thermal equilibrium when it is in thermal contact with its surroundings and when macroscopic changes have ceased to occur. Q represents heat transfer, which is the transfer of energy to a system, bringing about the same changes as if work were done, but without the performance of work; energy transfer by means of heat is associated with a temperature difference across the system boundary. Work, $W = \int \vec{F} \cdot d\vec{r}$, is the action of a force over a distance, with at least a small component of the force directed along the displacement; the most common type of work in thermodynamics

is $\int p\, dV$, due to the expansion or contraction of a system. ΔU is defined by the first law as equal to $Q - W$; ΔU is a state function that depends only on the internal system properties and not on the path between two states.

(c) $\Delta V = (0.100)(M_{water}/\rho_{water})$,

$$W = p\,\Delta V = \frac{(1.00 \times 10^5 \text{ Pa})(0.100)(1.00 \text{ kg})}{10^3 \text{ kg/m}^3} = +10.0 \text{ J}.$$

Since $\Delta U = Q - W$, ΔU is larger in magnitude than the heat of fusion by 10 J.

MASTERY TEST GRADING KEY - Form C

1. Solution: For a constant-volume gas thermometer:

$T \propto p$. Therefore

$$T = \frac{(92°C + 273.15)(0.60 \times 10^5\ Pa)}{1.60 \times 10^5\ Pa} = 137\ K.$$

2. Solution: $A = 10^{-2}\ m^2$, $\Delta m/\Delta t = 4.6 \times 10^{-3}$ kg/s,

$L_{vap} = 2.30 \times 10^6$ m, $\Delta Q/\Delta t = kA(\Delta T/\Delta x)$

$$\frac{\Delta T}{\Delta x} = \frac{\Delta Q/\Delta t}{kA} = \frac{(2.30 \times 10^6)(4.6 \times 10^{-3})}{(4.5 \times 10^2)(10^{-2})} = 2.35 \times 10^3\ K/m.$$

3. Solution: (a)

$\Delta Q_{12} = -10.0$ J $\quad 1 \rightarrow 2$

$\Delta Q = 0 \quad 2 \rightarrow 3 \quad \Delta U = 200$ J.

$\Delta Q = 0 \quad 3 \rightarrow 4$

$V_1 = 3.00 \times 10^{-2}\ m^3$, $V_2 = 2.00 \times 10^{-2}\ m^3$,

$V_3 = 1.00 \times 10^{-2}\ m^3$, $p_2 = 4.0 \times 10^4\ N/m^2$.

$$\text{Work} = \text{area} = (1/2)(V_2 - V_1)(p_2) + p_2(V_3 - V_2)$$

$$= (1/2)(1.00 \times 10^{-2})(4.0 \times 10^4) + (4.0 \times 10^4)(1.00 \times 10^{-2})$$

$$= 2.00 \times 10^2 + 4.0 \times 10^2 = 6.0 \times 10^2$$

Work = 600 J.

(b) Process $1 \rightarrow 2$: $\Delta Q = -10.0$ J, $\Delta W = +200$ J,

$\Delta U_{12} = \Delta Q - \Delta W = -10 + 200 = +190$ J.

Process $2 \rightarrow 3$: $\Delta Q = 0$, $\Delta W = -400$ J, $\Delta U_{23} = \Delta Q - \Delta W = 0 + 400 = +400$ J.

Process $3 \rightarrow 4$: $\Delta Q = 0$, $\Delta W = 0$, $\Delta U_{34} = 0$. $\Delta U_j = 200$ J (juice lost).

Therefore, $\Delta U_{14} = +190 + 400 - 200 = +390$ J.

STUDY GUIDE

KINETIC THEORY OF GASES

INTRODUCTION

As you read this sentence you will experimentally demonstrate the general gas law at least once by breathing in and out. As you expand the volume in your lungs, the pressure drops and air comes in; as you decrease the lung volume the pressure rises and air goes out. Pressure (p) and volume (V) are related; at constant temperature, pV = const.

This relation, called Boyle's law, was well established before the atomic theory of matter was accepted. In this module you will learn how to apply much of your knowledge of Newton's laws, kinetic energy, momentum, and elastic collisions to molecular motion. The fact that you can derive the macroscopic general gas law from Newtonian mechanics applied to molecules should give you further insight into the remarkable simplification Newton wrought in our understanding of matter. You will show that his laws hold equally well for masses 10^{27} times smaller and dimensions 10^{10} times smaller than the blocks and inclined planes we have discussed in previous modules. Newton's laws work from planetary physics to microphysics, an astronomical range!

To be sure, we have overstated the case in order to make a point. The behavior of an ideal gas is just that, idealized, and real gases only approximately behave this way. At the molecular level the fundamental "graininess" or quantization of momentum and energy as well as matter becomes important so that kinetic theory is also an idealization of nature.

PREREQUISITES

Before you begin this module, you should be able to:	Location of Prerequisite Content
*Solve mechanical problems using Newton's second law in the form: force = time rate of change of momentum (needed for Objective 3 of this module)	Newton's Laws Module
*Solve collision problems in which kinetic energy and momentum are conserved (needed for Objective 3 of this module)	Collisions Module
*Define the terms temperature, heat, and specific heat (needed for Objective 1 of this module)	Temperature, Heat, and Thermodynamics Module
*Differentiate a given polynomial (needed for Objectives 3 and 5 of this module)	Calculus Review
*Find the total rotational energy of a rigid body (needed for Objective 5 of this module)	Rotational Dynamics Module

LEARNING OBJECTIVES

After you have mastered the content of this module, you will be able to:

1. Ideal gas - macroscopic - Define an ideal gas from a macroscopic point of view using the equation of state, $pV = nRT$, and use this equation to solve problems involving the relations among the macroscopic variables p, V, n, and T.
2. Ideal gas - microscopic - Give the six postulates used in this module to define the microscopic kinetic-theory model of an ideal gas.
3. Kinetic theory - Solve problems based on the steps involved in the kinetic-theory derivation of the macroscopic pressure. These problems will be based on the relationships among pressure, force, velocity, momentum, and the number of particles.
4. Temperature - molecular speed - Use the definition of temperature in the kinetic model to show that the kinetic-theory pressure equation $pV = NmV_{rms}^2/3$ is equivalent to the equation of state; or solve problems involving the relationship among T, m, and v_{rms}.
5. Equipartition of energy - State the law of equipartition of energy and apply it to the calculation of C_v for gases; or apply it to the calculation of molecular speeds in containers holding a mixture of gases.

GENERAL COMMENTS

1. Order of Presentation

Kinetic theory may be defined as the statistical application of the laws of mechanics to assemblies of atoms using relatively simple averaging techniques. More advanced techniques are utilized in a branch of physics called statistical mechanics. This module introduces the kinetic theory of gases in the following order: (1) an ideal gas is defined from the macroscopic point of view as a gas that exactly follows the equation of state; (2) a list of postulates is given for a kinetic theory of gases from a microscopic point of view; (3) using these postulates, the macroscopic quantity pressure is calculated from the kinetic theory; and (4) by making an identification of temperature in terms of molecular kinetic energy, the macroscopic equation of state is derived from a molecular model using kinetic theory. Not all the texts handle the material in this order; however, if you will read the material assigned with the above outline in mind, you should have no difficulty with this module.

2. Postulates for the Kinetic Theory

Objective 2 asks you to list the assumptions that define the microscopic model of an ideal gas. Various texts list these in various orders and forms. The following list is given in terms of short phrases that will be easier to reproduce on the Mastery Test. Your written answers need not include more than this, but you may be asked to give more detail orally.

a. Large number of particles. The direction and speed of any molecule may change abruptly on collision, and any particular molecule will follow a zigzag path. However, the very large number of particles ensures a distribution of molecular velocities and randomness of the motion.

b. Negligible particle volume. The molecules of a gas are, on the average, separated by distances that are large compared with the molecular diameters. We assume the molecules have negligible volume compared with the volume of the container.

c. Random motion. The molecules move in all directions with equal probability and with a variety of speeds. The center of mass of the gas as a whole remains at rest, the total linear momentum being zero.

d. No forces except during collisions. Between collisions the molecules are free of forces and move with a constant speed. A collision takes place when one molecule is within the short-range intermolecular force of a second molecule. The duration of any collision is assumed to be very small compared with the time between collisions. Thus, although a pair of molecules will lose kinetic energy and gain potential energy during a collision, the potential energy can be ignored because a molecule spends a negligible fraction of its time in collisions.

e. Elastic collisions. The collisions between molecules are perfectly elastic.

f. Newtonian mechanics applies. In computing the properties of the motion, we assume that Newtonian mechanics works at the microscopic level. This assumption will stand or fall depending on whether or not the experimental facts it predicts are correct.

3. Temperature Dependence

Some texts do not place great emphasis on the result that the internal energy of an ideal gas, U, as derived using the kinetic theory, depends only on the temperature. The importance of this result will be seen in other modules (Second Law and Entropy).

STUDY GUIDE: Kinetic Theory of Gases 4(B 1)

TEXT: Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

Read the General Comment 1. This will make clearer the reason for the selection of text readings. Read Chapter 15, Sections 15.1, 15.2, and 15.4 and Chapter 16, Sections 16.4 and 16.7. Bueche does not introduce the equation of state, $pV = nRT$, as a macroscopic law, but proceeds to calculate it from kinetic theory. This accounts for the arrangement of readings with respect to the objectives of this module.

The author gives the assumptions of the kinetic theory required in Objective 2 in narrative form. For your convenience these are listed in General Comment 2. Read General Comment 3. Study Problems A through I. Then solve the Problems J through O. If you need more practice, you may work some of the optional Additional Problems

Try the Practice Test.

BUECHE

Objective Number	Readings	Problems with Solutions		Assigned Problems	Additional Problems
		Study Guide	Text	Study Guide	
1	General Comment 1, Sec. 15.4	A, B	Illus.[a] 15.1; 15.2	J, K	Chap. 15, Probs. 6, 7, 11-13
2	Sec. 15.1, General Comment 2				
3	Sec. 15.2	C, D		L, M	
4	Sec. 15.4, General Comment 3	E, F	Illus. 15.3	N, O	Chap. 15, Probs. 3, 4, 8, 9
5	Secs. 16.4, 16.7	G, H, I	Illus. 16.2	P, Q	Chap. 16, Probs. 10, 14, 27

[a] Illus. = Illustration(s).

STUDY GUIDE: Kinetic Theory of Gases 4(HR 1)

TEXT: David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974)

SUGGESTED STUDY PROCEDURE

Read Chapter 20, Sections 20-1 through 20-7. In your reading of Section 20-6 on the specific heat, you should read the derivation of C_p as well as of C_v. Although only C_v will be studied in this unit, the next module (Second Law and Entropy) requires the definition of C_p. Your text uses the symbol μ for the number of moles, whereas we have used the more common symbol n ($\mu \equiv n$). Read General Comments 1 through 3. Study Problems A through I. Then solve problems J through Q. If you need more practice, you may work some of the optional Additional Problems from your text.

Try the Practice Test.

HALLIDAY AND RESNICK

Objective Number	Readings	Problems with Solutions		Assigned Problems	Additional Problems
		Study Guide	Text	Study Guide	(Chap. 20)
1	General Comment 1, Sec. 20-2	A, B	Ex.[a] 1, 2	J, K	2, 3, 4
2	General Comment 2, Sec. 20-3				
3	Sec. 20-4	C, D	Ex. 3, 4	L, M	13
4	Sec. 20-5, General Comment 3	E, F		N, O	14, 15, 16, 18, 21, 23, 24
5	Secs. 20-6, 20-7	G, H, I	Ex. 6	P, Q	22, 23, 43, 44

[a]Ex. = Example(s).

STUDY GUIDE: Kinetic Theory of Gases 4(SZ 1)

TEXT: Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition

SUGGESTED STUDY PROCEDURE

Read the General Comment 1. This will make clearer the reason for the selection of the text readings. Read Chapter 18, Sections 8-1 and 8-2, and Chapter 20, Sections 20-1 to 20-4 and 20-7. Objective 2 deals with the specific assumptions in the kinetic-theory derivation of the equation of state. The text gives these in narrative style in Section 20-3 (pp. 287, 288). For your convenience in mastering Objective 2, the assumptions are listed in General Comment 2.

Read General Comment 3. Then study Problems A through I before solving Problems J through Q. If you need more practice, you may wish to work some of the optional Additional Problems from your text.

Try the Practice Test.

SEARS AND ZEMANSKY

Objective Number	Readings	Problems with Solutions		Assigned Problems	Additional Problems
		Study Guide	Text	Study Guide	
1	General Comment 1, Secs. 18-1, 18-2	A, B	Sec. 18-2, Ex.[a] 1	J, K	18-1 to 18-12, 20-1 to 20-3
2	General Comment 2, Sec. 20-3				
3	Secs. 20-1 to 20-3	C, D		L, M	20-4
4	Sec. 20-3, General Comment 3	E, F	Sec. 20-3, Ex. 1, 2, 3, 4	N, O	20-5, 20-8
5	Secs. 20-4, 20-7	G, H, I		P, Q	20-6, 20-7

[a]Ex. = Example(s).

STUDY GUIDE: Kinetic Theory of Gases 4(WS 1)

TEXT: Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 1

SUGGESTED STUDY PROCEDURE

Read the General Comment 1. This will make clearer the reason for the selection of readings in your text. Read Chapter 18, Sections 18-5 and 18-6, Chapter 19, Sections 19-1 to 19-3 and 19-5, General Comment 2, and Chapter 20, Section 20-4. You may already have read Section 19-5 on specific heat in relation to the module Temperature, Heat, and Thermodynamics; however, the emphasis now is on the influence of extra degrees of freedom introduced by rotation and vibration.

Read General Comment 3. Then study Problems A through I before solving Problems J through Q. If you need more practice you may work some of the optional Additional Problems from your text.

Try the Practice Test.

WEIDNER AND SELLS

Objective Number	Readings	Problems with Solutions		Assigned Problems	Additional Problems
		Study Guide	Text	Study Guide	
1	General Comment 1, Secs. 18-5, 18-6	A, B	Ex.[a] 18-1, 18-2	J, K	18-2, 18-11, 18-12, 18-14 to 18-18, 18-20
2	General Comment 2, Secs. 19-1, 19-2				
3	Sec. 19-2	C, D	Ex. p. 381	L, M	19-2, 19-5
4	Sec. 19-3, General Comment 3	E, F	Ex. 19-1	N, O	19-6 to 19-8, 19-12
5	Secs. 19-5, 20-4	G, H, I		P, Q	19-19, 19-20

[a]Ex. = Example(s).

PROBLEM SET WITH SOLUTIONS

A(1). Show that the kinetic-theory model of an ideal gas obeys Avogadro's law: If two containers of gas have the same volume, pressure, and temperature, then there are equal numbers of molecules in them.

Solution

$$pV = nRT = (N/N_0)kN_0T = NkT$$

since $n = N/N_0$ and $R = kN_0$. Therefore,

$$N = pV/kT.$$

It is given that

$$p_1V_1/kT_1 = p_2V_2/kT_2 = \text{const},$$

therefore

$$N = pV/kT = \text{const}.$$

B(1). A tank having a volume of 30.0 ℓ contains nitrogen (N_2) gas at 20° C and at a gauge pressure of 3.00×10^5 Pa. The tank's valve is opened momentarily, and some N_2 escapes. After the valve is closed and the gas has returned to room temperature (20° C), the tank's pressure gauge reads 2.40×10^5 Pa. How much N_2 leaked out?

Solution

Before: $p_1V_1 = n_1RT_1$, After: $p_2V_2 = n_2RT_2$. We wish to find

$$n_1 - n_2 = p_1V_1/RT_1 - p_2V_2/RT_2.$$

But $T_1 = T_2$ and $V_1 = V_2$, therefore,

$$n_1 - n_2 = \frac{V}{RT}(p_1 - p_2) = \frac{(3.00 \times 10^{-2}\ \text{m}^3)(6.00 \times 10^4\ \text{Pa})}{(8.31\ \text{J/g mol K})(293\ \text{K})} = 7.4 \times 10^{-1}\ \text{mol}.$$

But since 1 mol of N_2 = 28.0 g = 0.0280 kg,

$$m = n(0.0280) = 7.4 \times 10^{-1} \times 2.80 \times 10^{-2} = 2.10 \times 10^{-2}\ \text{kg}.$$

C(3). Precisely one million molecules strike a surface with an area of 1.00×10^{-2} m^2 in one second and bounce elastically. Each of these molecules strikes at an angle of 60° from the normal, each has a mass of 3.30×10^{-27} kg, and the average speed of these million molecules is 330 m/s (the speed of sound at normal conditions).

(a) What is the magnitude of the total momentum of these molecules before they strike the surface? How large is its component perpendicular to the surface?

(b) What average pressure do these molecules exert on this surface during the one second they are striking it?

(c) Normal atmospheric pressure is about 10^5 Pa; why is this so different from your answer to (b)?

Solution

(a) The momentum of one molecule is mV; the momentum of 10^6 molecules is

$$Nmv = (10^6)(3.30 \times 10^{-27}\ \text{kg})(330\ \text{m/s}) = 1.10 \times 10^{-18}\ \text{kg m/s}.$$

The component perpendicular to the surface is

$$Nmv \cos 60° = 0.55 \times 10^{-18}\ \text{kg m/s}.$$

(b) The pressure is force per unit area. The pressure on the wall is

$$p = \frac{-Nd(mv)}{A\ dt} = \frac{-N(mv_{x_f} - mv_{x_i})}{At} = \frac{-N(-mv \cos 60° - mv \cos 60°)}{A(1.00\ \text{s})}$$

$$= \frac{2Nmv \cos 60°}{A} = \frac{2(1.10 \times 10^{-18}\ \text{kg m/s})(0.50)}{(10^{-2}\ \text{m}^2)(1.00\ \text{s})} = 1.10 \times 10^{-16}\ \text{Pa}.$$

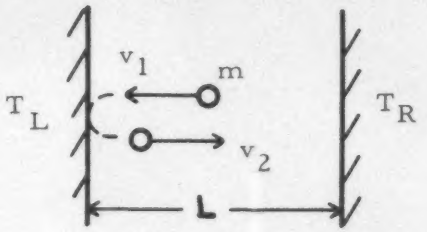
(c) There are approximately 3×10^{25} molecules per cubic meter in a gas at atmospheric pressure: the number hitting the wall per second is

$$(1/3)pAv = (3 \times 10^{25}\ \text{molecules}/3)(10^{-2}\text{m}^2)(3 \times 10^3\ \text{m/s}).$$

This equals about 10^{27} collisions per second, which is 10^{21} times more than we assumed. Therefore,

$$p_{atm} = (10^{21})(1.10 \times 10^{-16}\ \text{Pa}) \simeq 10^5\ \text{Pa}.$$

D(3). Assume that a particle bounces normally off a heated surface with a kinetic energy that is in thermal equilibrium with the surface temperature regardless of its approach energy. Calculate the average force F produced on the left-hand plate by a single molecule of mass m in bouncing back and forth.



Solution

$$F = \left(\frac{\text{number of collisions}}{\text{second}}\right)(\text{change in momentum of wall}) = \left(\frac{1.00\ \text{s}}{\text{round-trip time}}\right)\left(\frac{mv_2 + mv_1}{1.00\ \text{s}}\right).$$

The round-trip time is the sum of right to left and the return:

$$t = L/v_1 + L/v_2.$$

Therefore,

$$F = m(v_2 + v_1)v_1v_2/(Lv_2 + Lv_1) = mv_1v_2/L.$$

Note that this reduces to

$$F = mv^2/L \quad \text{if } v_1 = v_2.$$

From the kinetic definition of temperature

$$(1/2)kT = (1/2)mv_x^2,$$

we can evaluate v in terms of T:

$$v_1 = (kT_R/m)^{1/2}; \quad v_2 = (kT_L/m)^{1/2}.$$

Thus

$$F = (mk/Lm)(T_RT_L)^{1/2} = (k/L)(T_RT_L)^{1/2}.$$

E(4). Escape velocity from the Moon is approximately 5310 miles per hour = 2.37×10^3 m/s. At certain spots on the surface of the Moon, the temperature of the surface material reaches 500 K. Suppose a molecule of hydrogen

($m = 3.33 \times 10^{-27}$ kg) were to hit the surface of the Moon and reach a speed equal to the root-mean-square (rms) speed for this temperature. What would happen to this molecule?

Solution

$$(mv^2/2)_{av} = (3/2)kT, \quad (v^2)_{av} = 3kT/m,$$

$$v_{rms} = m/3kT)^{1/2} = \left[\frac{3(1.38 \times 10^{-23}\ J/K)(500\ K)}{3.33 \times 10^{-27}\ kg}\right]^{1/2} = \left[\frac{3(1.38 \times 5 \times 10^{6}\ m^2/s^2)}{3.33}\right]^{1/2}$$

$$= 2.49 \times 10^{3}\ m/s.$$

Since this speed is faster than escape velocity, the hydrogen molecule would escape from the Moon. Although other more massive molecules would not achieve escape velocity as easily, enough of them have so that the Moon now has no atmosphere.

F(4). The kinetic-theory derivation of the macroscopic pressure gave the result

$$p = (Nmv^2_{rms})/3V.$$

It is easy to identify the average kinetic energy of the gas in this formula and reasonable to guess that temperature, which we know is related to heat energy, is proportional to the kinetic energy. Show that if

$$T = \frac{1}{2}\left(\frac{Mv^2_{rms}}{3nR/2}\right),$$

the pressure formula above reduces to the equation of state for an ideal gas.

Solution

Rearranging the pressure equation, we find

$$pV = \frac{Nmv^2_{rms}}{3} = \frac{Mv^2_{rms}}{3} = \frac{2}{3}\left(\frac{1}{2}Mv^2_{rms}\right).$$

From the definition of T, we see that $(1/2)Mv^2_{rms} = (3/2)nRT$. Thus $pV = nRT$.

G(5). An air-table model of an ideal gas has pucks with masses of 2.00×10^{-2} kg and 8.0×10^{-2} kg. It is found that the 8.0×10^{-2} kg pucks have an rms speed of 0.50 m/s. An air table provides a level, nearly frictionless surface. The pucks, riding on a cushion of air, make nearly elastic

collisions with each other and with a taut wire fence at the boundaries.
(a) Approximately what will be the rms speed of the 2.00×10^{-2} kg pucks?
(b) The motion of the pucks in this model is, of course, provided by making the wire "fence" around the edges oscillate back and forth. However, if we raised the temperature of the pucks and the air table sufficiently, this motion of the pucks would occur naturally, as a thermal motion. (Assume the pucks and air table can stand the high temperature.) What temperature would be needed for this?

Solution

(a) By the equipartition theorem, each degree of freedom has the same energy [(1/2)kT] for each of the two translational axes, and therefore kT for the sum of the two. The "temperature" must be the same for the small pucks, therefore

$$(1/2)m_1 v^2_{(rms)1} = (1/2)m_2 v^2_{(rms)2}$$

or

$$v_{(rms)2} = \left(\frac{m_1}{m_2}\right)^{1/2} v_{(rms)1} = \left(\frac{8.0}{2.00}\right)^{1/2}(0.50) = 1.00 \text{ m/s}.$$

(b) If the motion were cased by temperature,

$$T = \frac{1}{2k} m_1 v^2_{rms} = \frac{(8.0 \times 10^{-2} \text{ kg})(0.50 \text{ m/s})^2}{2(1.38 \times 10^{-23} \text{ J/K})} = 7.2 \times 10^{20} \text{ K}.$$

H(5). (a) State the equipartition theorem.
(b) What is the specific molar heat at constant volume for a diatomic molecule that can be represented as a rigid dumbbell?

Solution

(a) There is one degree of freedom associated with each independent coordinate of the system of particles. Each degree of freedom has the same mean energy at thermal equilibrium [(1/2)kT].

(b) There are three degrees of freedom for translational kinetic energy. For a dumbbell there are only two rotational degrees of freedom, since for point masses the angle of rotation about an axis through the two masses is not a physical independent coordinate ($I_y = I_z$; $I_x = 0$). Since there are a total of five degrees of freedom,

$$U = (5/2)NkT,$$

$$C_v = \frac{dU}{dt} = \frac{d}{dt}\left(\frac{5}{2}N_0kT\right) = \frac{5}{2}N_0k = \frac{5}{2}R = \frac{5}{2}(8.31\ \text{J/mol K}) = 20.8\ \text{J/mol K}.$$

I(5). Calculate the specific heat of a three-dimensional solid.

Solution

Since a solid has its atoms in fixed locations the atoms are not free to translate. Neither do atoms have rotational degrees of freedom because of the structure of a solid. However, they do have vibrational degrees of freedom in each of three dimensions. Thus there are three degrees of freedom for vibrational potential energy and three degrees of freedom for vibrational kinetic energy for a total of six degrees of freedom. Thus

$$V = 6(1/2)N_0kT$$

for a mole of the solid, and

$$C_v = (dU/dt)_v = 3N_0k = 3R = 3(8.31\ \text{J/mol K}) = 24.9\ \text{J/mol K}.$$

Problems

J(1). An evacuated tank with a capacity of 40 ℓ is filled with helium at room temperature (20.0°C). The mass of the tank is thereby increased by 0.150 kg. What then is the pressure registered on the tank's pressure gauge?

K(1). Imagine a cube of gas kept at a temperature of 300 K and a pressure of 1.00×10^5 Pa. If this cube contains a number of molecules equal to the population of the world (4 000 000 000 people), what is the length of one of its edges?

L(3). The mass of a H_2 molecule is 3.32×10^{-27} kg. If 1.00×10^{23} hydrogen molecules per second strike $2.00 \times 10^{-4}\text{m}^2$ of wall at an angle of 45° with the normal when moving with a speed of 10^3 m/s, what pressure do they exert on the wall?

M(3). The temperature at the Sun's center is estimated to be about 14×10^6 K, whereas the density there is about 1.00×10^5 kg/m^3. Assuming the protons that constitute most of the Sun's core act like a perfect gas even at these very high densities, find the pressure of the proton gas at the Sun's core ($m_p = 1.67 \times 10^{27}$ kg).

N(4). The fragments from uranium atoms that have undergone unclear fission have an average kinetic energy of 1.10×10^{-11} J. What would be the approximate temperature of a gas consisting of such fission fragments?

O(4). Show that the kinetic-theory pressure equation is equivalent to the equation of state of an ideal gas if

$$T = NK \qquad T = \frac{NK}{3nR/2}, \qquad \text{where } K = (1/2)mv_{rms}^2.$$

P(5). Assuming air to be composed of 78% nitrogen (N_2) and 22.0% oxygen (O_2) by weight, what should be the value of molar C_V for air?

Q(5). A flask contains a mixture of mercury vapor, neon, and helium. Compare (a) the average kinetic energies of the three types of atoms; and (b) the rms speeds.

Solutions

J(1). 23.0×10^5 Pa.

K(1). 5.5×10^{-6} m.

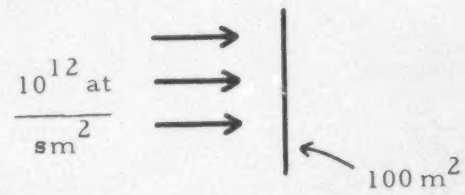
L(3). 2400 Pa.

M(3). 1.20×10^{16} Pa.

N(4). 5.3×10^{11} K.

O(4). $T = \frac{NK}{(3/2)nR} = \frac{N(1/2)mv_{rms}^2}{(3/2)nR} = \frac{Nmv_{rms}^2}{3nR}$; $nRT = Nmv_{rms}^2/3$. But $pV = Nmv_{rms}^2/3$, from the pressure equation. Thus $pV = nRT$.

P(5). 0.72 J/mol K.



PRACTICE TEST

1. (a) Give a macroscopic definition of an ideal gas.

   (b) A bubble of air rises from the bottom of a lake, where the pressure is 3.03×10^5 Pa, to the surface, where the pressure is 1.00×10^5 Pa. The temperature at the bottom of the lake is 7.0°C, and the temperature at the surface is 27.0°C. What is the ratio of the volume of the bubble as it reaches the surface to the volume of the bubble at the bottom?

2. There is a flow of gas radially outward from the Sun into space called the "solar wind." In the vicinity of the Earth the flow is primarily hydrogen atoms with a speed of 3.00×10^4 m/s. If the flux of atoms is 10^{12} atoms per second per square meter, calculate the acceleration of a 100-m^2 thin aluminum foil sail, normal to the flux of atoms, which has a mass of 10^{-2} kg ($M_H = 1.67 \times 10^{-27}$ kg). (Assume elastic collisions.) (See figure above.)

3. Give six statements that define the microscopic kinetic-theory model of an ideal gas.

4. In some nuclear reactions, it is desirable to have "thermal neutrons," i.e., a large number of neutrons with a temperature T = 27.0°C.
   (a) What is your best estimate of the kinetic energy of such a neutron?
   (b) What is your best estimate of its speed?

5. Ammonia (NH_3) is a polyatomic gas. Use the kinetic theory to derive the value of C_v for ammonia.

Practice Test Answers

1.(a) An ideal gas is one that satisfies the equation of state pV = nRT.

(b) At the bottom: $p_1V_1 = nRT_1$; at the surface: $p_2V_2 = nRT_2$. Since n is a constant, $p_2V_2/p_1v_1 = T_2/T_1$, or

$$\frac{V_2}{V_1} = \frac{T_2}{T_1}\,\frac{p_2}{p_1} = \frac{300\ K}{280\ K}\,\frac{3.03 \times 10^5\ Pa}{1.0C \times 10^5\ Pa} = 3.24.$$

2. $F = ma = \frac{dp}{dt} = \dot{A}(2mV)\#/s\ m = A(\frac{\#}{s})(2mV)$,

$$a = \frac{A}{M}\,(\tfrac{\#}{s})(2mV) = (\frac{10^2\ m^2}{10^{-2}\ kg})(\frac{10^{12}}{m^2\ s})2(1.67 \times 10^{-27}\ kg)(3.00 \times 10^4\ m/s) = 1.00 \times 10^{-6}\ m/s^2.$$

3. Large numbers of particles, negligible particle volume, random motion, no forces except during collisions, elastic collisions, Newtonian mechanics applies.

4.(a) $(KE)_{av} = \frac{1}{2}N^{-2}_{rms} = \frac{3}{2}kT = \frac{3}{2}(1.38 \times 10^{-23}\ J/K)(300\ K) = 6.2 \times 10^{-21}\ J.$

(b) $v_{rms} = [2(KE)_{av}/m]^{1/2} = [2(6.2 \times 10^{-21}\ J)/(1.67 \times 10^{-27}\ kg)]^{1/2} = 2.70 \times 10^3\ m/s.$

5. Ammonia has three translational degrees of freedom and three rotational ones:

$K_{trans} = (3/2)N_0kT; \quad K_{rotat} = (3/2)N_0kT;$

$U = K_{trans} + K_{rotat} = 3N_0kT; \quad C_v = dU/dt = 3N_0k = 3R.$

Date_____

KINETIC THEORY OF GASES pass recycle

Mastery Test Form A 1 2 3 4 5

Name _____ Tutor _____

1. (a) Give a macroscopic definition of an ideal gas.
   (b) How many molecules per cubic meter are there in a radio-tube vacuum? The pressure is about 10^{-9} atm (1 atm = 1.01×10^5 Pa). Assume T = 300 K.

2. List three of the six postulates that define an ideal gas in the microscopic model.

3. Professor M.V. Square handed to his secretary the following kinetic-theory derivation of the pressure on the wall of a cubical box of side ℓ, containing N molecules in random motion. The secretary left blanks when she could not decipher his handwriting. Fill in the blanks.

   "Suppose a particle strikes the wall. The change in _____/collision = _____. The change in _____ of the wall (area = ℓ^2) = _____ by conservation of _____. After a time t, the particle should again strike the same wall. This time t = _____, so that the number of collisions per second with the wall is _____. The average force is given by the rate of change of _____ per unit _____. Therefore, for one particle F = _____. We now add up the forces for N molecules and divide by _____ to get

   $$p = (_____) \sum_{i=1}^{N} _____.$$

   However, the general definition of the average is

   $$\sum_{i=1}^{N} \frac{x_i}{N} = x_{av}.$$

   Therefore,

   $$p = (_____)(v_x^2)_{av}.$$

   Now

   $$v^2 = v_x^2 + v_y^2 + v_z^2,$$

   and since the velocities are assumed random

   $$(v_x^2)_{av} = (_____)(v^2)_{av} \quad \text{and} \quad p = _____."$$

(Continued on next page)

KINETIC THEORY OF GASES

Mastery Test Form A

4. The kinetic theory model of an ideal gas contains many predictions. For instance:
(a) Show what happens to the pressure when the speed of every molecule is doubled, and the volume is kept unchanged.
(b) What does this doubling do to the internal energy of the gas?
(c) What does this indicate about the relationship between the pressure and the temperature, in general, when the volume is kept constant? (You should come up with what is known as Gay-Lussac's or Charles' law. Recall that in the kinetic-theory model, the temperature is defined in terms of the internal energy - what is that definition?)

5. (a) State the equipartition theorem.
(b) Equal masses of hydrogen and oxygen are mixed. What is the ratio of numbers of molecules? What is the ratio of average kinetic energy per molecule? What is the ratio of average speeds? What is the ratio of pressures exerted on the walls? ($M_{H_2} = 2$, $M_{O_2} = 32$.)

Date _____

KINETIC THEORY OF GASES pass recycle

Mastery Test Form B 1 2 3 4 5

Name _____ Tutor _____

1. (a) Give a macroscopic definition of an ideal gas.
   (b) An automobile tire has a volume of 1.60×10^{-2} m^3 and contains air at a pressure of 2.70×10^5 Pa (approximately 24 lb/in.^{2} gauge pressure) when the temperature is 0.00°C. What is the pressure of the air in the tube when its temperature rises to 27.0°C and its volume increases to 1.63×10^{-2} m^3?

2. Give three of the six postulates that define a microscopic ideal gas.

3. Suppose that one molecule occupies a cubical box all by itself. Assume the walls of the box are perfectly smooth, so that the molecule bounces away from any wall at the same angle at which it struck the wall. Originally, the molecule has the speed v_0, and exerts the <u>average</u> pressure p_0 on the right-hand wall. But now, by some cunning device, we manage to accelerate this molecule to twice its original speed (i.e., to $2v_0$) without changing its direction of motion. How large is the average force it now exerts on the right-hand wall?

4. Find the rms speed of helium atoms at 40°C given that the rms speed of oxygen molecules at 0.00°C is 460 m/s. The molecular weights are O_2 = 32.0 g/mol; He = 4.0 g/mol.

5. (a) State the equipartition theorem.
   (b) Show that the kinetic-theory model of an ideal gas obeys Dalton's law of partial pressures: When several different gases occupy the same container, the pressure they exert on the container is the sum of the pressures that each one would exert if it alone occupied the container.

Date _____

KINETIC THEORY OF GASES

pass recycle

Mastery Test Form C

1 2 3 4 5

Name _____ Tutor _____

1. (a) Give a macroscopic definition of an ideal gas.
   (b) While preparing a sealed-off 2.00×10^{-5} m^3 tube at low temperatures, one nitrogen drop of mass 5.6×10^{-5} kg is accidentally sealed off in the tube. Find the nitrogen (N_2) pressure within the tube when the tube warms to 300 K (M_{N_2} = 14.0 g/mol).

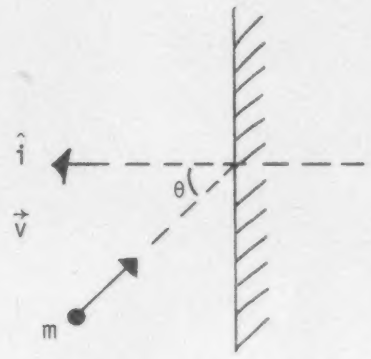
2. List three of the six postulates that define an ideal gas in the microscopic model.

3. A stream of gas molecules (mass m) is incident upon a wall as in Figure 1. If the collisions are elastic, the velocity of the molecules is $\vec{v}$, and the normal to the wall is $\hat{i}$, what pressure is exerted on the wall by the molecules if N molecules strike the wall per unit time per unit area?

4. (a) Determine the average value of the kinetic energy of the molecules of an ideal gas at 0.00°C.
   (b) What is the kinetic energy per mole of an ideal gas at these temperatures?

5. (a) State the equipartition theorem.
   (b) The mass of a gas molecule can be computed from the specific heat at constant volume. Take C_V = 0.314 J/kg K for argon and calculate (i) the mass of an argon atom, and (ii) the atomic weight of argon.

Figure 1



MASTERY TEST GRADING KEY - Form A

1. Solution: (a) An ideal gas is one that exactly obeys the equation of state $pV = nRT$. (b) $pV = nRT$. We want to transform to a molecular basis since we need molecules per cubic meter = N/V:

$$n = N/N_0 \quad \text{and} \quad R = N_0 k,$$

where N is the number of molecules, N_0 is the number of molecules per mole, and k is Boltzmann's constant. Therefore, $pV = NkT$ or

$$\frac{N}{V} = \frac{p}{kT} = \frac{(1.01 \times 10^5 \text{ Pa}) \times 10^{-9}}{(1.38 \times 10^{-23} \text{ J/K})(300 \text{ K})} = 2.44 \times 10^{16} \text{ molecules/m}^3.$$

2. What To Look For: Student should have any three of the following.

Solution: (1) Large number of particles; (2) Negligible volume; (3) Random motion; (4) No forces except during collision; (5) Elastic collisions; (6) Newtonian mechanics applies.

3. Solution:

"Suppose a particle strikes the wall. The change in MOMENTUM/collision = $-2mv_x$. The change in MOMENTUM of the wall (area = ℓ^2) = $+2mv_x$ by conservation of MOMENTUM. After a time t, the particle should again strike the same wall. This time $t = 2\ell/v_x$, so that the number of collisions per second with the wall is $v_x/2\ell$. The average force is given by the rate of change of MOMENTUM per unit TIME. Therefore, for one particle

$$F = dp/dt = (2mv_x)(v_x/2\ell) = mv_x^2/\ell.$$

We now add up the forces for N molecules and divide by AREA to get

$$p = \frac{m}{A\ell} \sum_{i}^{N} v_{x_i}^2.$$

However, the general definition of the average is

$$\sum_{i=1}^{N} \frac{x_i}{N} = x_{av}.$$

Therefore,

$$p = (mN/A\ell)(v_x^2)_{av}.$$

Now,

$$v^2 = v_x^2 + v_y^2 + v_z^2,$$

and since the velocities are assumed random

$$(v_x^2)_{av} = \underline{(1/3)}(v^2)av \quad \text{and} \quad p = \underline{(mN/3A\ell)(v^2)_{av} = (mN/3V)v_{rms}^2}.$$

4. Solution: (a) $p = (mN/3A\ell)v_{rms}^2$.

If we double v_{rms}^2, p is quadrupled.

(b) $U = N[(1/2)mv_{rms}^2]$. If we double v_{rms}^2, U is quadrupled.

(c) The temperature is proportional to the internal energy U. In fact, to fit the macroscopic and microscopic pV expressions $pV = nRT = NkT$ and

$$pV = (mN/3)v_{rms}^2 = (2/3)N(\tfrac{1}{2}mv_{rms}^2),$$

T must equal $(2/3k)[(1/2)mv_{rms}^2]$ or $(3/2)kT = (1/2)mv_{rms}^2$. If V = constant and $p = NkT/V = \text{const}\ T$, therefore $p/T = \text{const}$, which is Gay-Lussac's law.

5. Solution: (a) The available energy depends only on temperature and distributes itself in equal shares to each of the independent ways in which the molecule can absorb energy.

(b) If the total masses of H_2 and O_2 are equal:

$$m_{H_2}N_{H_2} = m_{O_2}N_{O_2};$$

where m is the mass of one atom or

$$(M_{H_2}/N_0)N_{H_2} = (M_{O_2}/N_0)N_{O_2}$$

since $m = M/N_0$. Therefore

$$N_{H_2}/N_0 = M_{O_2}/M_{H_2} = 32/2 = 16.$$

By the equipartition theorem

$$(\tfrac{1}{2}m_{H_2}v_{H_2}^2) = (\tfrac{1}{2}m_{O_2}v_{O_2}^2.$$

Therefore the ratio of kinetic energies is 1 and the ratio of the speeds

$$v_{H_2}/v_{O_2} = \sqrt{M_{O_2}/M_{H_2}} = \sqrt{16} = 4.$$

The pressures are

$$p_{O_2} = (m_{O_2} N_{O_2}/3V)v_{O_2}^2, \qquad p_{H_2} = (m_{H_2} N_{H_2}/3V)v_{H_2}^2,$$

and dividing we find

$$\frac{p_{H_2}}{p_{O_2}} = \frac{M_{H_2}}{M_{O_2}}(16)(4) = (\frac{1}{16})(16)(4) = 4.$$

MASTERY TEST GRADING KEY - Form B

1. Solution: (a) An ideal gas is one that exactly follows the equation of state $pV = nRT$.
(b) When the tire is cold (condition 1) $p_1V_1 = nRT_1$. When hot (condition 2) $p_2V_2 = nRT_2$. Dividing we get

$$p_1V_1/p_2V_2 = T_1/T_2$$

or

$$p_2 = \frac{p_1V_1T_2}{V_2T_1} = \frac{(2.70 \times 10^5 \text{ Pa})(1.60 \times 10^{-2} \text{ m}^3)(273 + 27) \text{ K}}{(1.63 \times 10^{-2} \text{ m}^3)(273 \text{ K})} = 2.91 \times 10^5 \text{ Pa}$$

or

$$\frac{2.90 \times 10^5 \text{ Pa}}{6.9 \times 10^3 \text{ Pa lb}^{-1} \text{ in.}^2} = 42 \text{ lb/in.}^2 \text{ abs.} = 42 - 15 = 27 \text{ psi gauge.}$$

2. What To Look For: Student should have any three of the following.

Solution: (1) Large number of particles; (2) Negligible volume; (3) Random motion; (4) No forces except during collisions; (5) Elastic collisions; (6) Newtonian mechanics applies.

3. Solution: Originally

$$p_0 = \frac{\text{av. force}}{\text{area}} = \left(\frac{\text{number collisions}}{\text{unit time}}\right)\frac{(-\Delta p)}{\text{area}} = \left(\frac{v_0}{2\ell}\right)\left(\frac{2mv_0}{A}\right) = \frac{mv_0^2}{\ell A} = \frac{mv_0^2}{V}.$$

If v_0 is doubled,

$$p_1 = (2v_0/2\ell)(2m2v_0/A) = 4mv_0^2/V.$$

Therefore

$$p_1/p_0 = 4.$$

4. Solution: Here is how we will proceed:

$$mv_{rms}^2/2 = 3kT/2$$

represents the kinetic energy of a molecule of an ideal gas. From the information given on oxygen, i.e., T and v_{rms}, knowing k (Boltzmann's constant), we can calculate $m(O_2)$, which is the mass of an O_2 molecule. We know that a mole of a gas contains N_0 (Avagadro's number) molecules. Relative to carbon

this mole is assigned M (molecular weight) grams. Therefore a molecule has a mass of M/N_0; so $m(O_2) = M(O_2)/N_0$:

N_0 (molecules per mole) × m (mass of a molecule) = M (mass of a mole of gas).

We have $m(He) = m[m(O_2)/M(O_2)] = M(He)$.

Knowing the mass of the argon and helium molecules; we can then calculate v_{rms}:

$$v_{rms} = [3kT/m]^{1/2},$$

so

$$m(O_2) = \frac{3kT}{v_{rms}^2(O_2)} = \frac{(3)(1.381 \times 10^{-23})}{(460\ m/s)^2} \frac{J}{\text{molecule K}} (273.15\ K)$$

$$= \frac{(3)(1.381)(2.7315)}{(4.6)(4.6)\ m^2/s^2} \times \frac{10^{-25}\ kg\ m^2/s^2}{\text{molecule}} = 5.348 \times 10^{-26}\ \text{kg/molecule}.$$

$$M(He) = 5.348 \times 10^{-26}\ \text{kg/molecule}\ \frac{4}{32}\ \frac{g/mole}{g/mole} = 0.6685 \times 10^{-26}\ \text{kg/molecule},$$

$$v_{rms}(He) = [(3)(1.381 \times 10^{-23}\ \text{J/molecule K}) \frac{313.15\ K}{0.6685 \times 10^{-26}\ \text{kg/molecule}}]^{1/2}$$

$$= [1.9407 \times 10^6\ m^2/s^2]^{1/2} = 1393\ m/s.$$

5. Solution: (a) The available energy depends only on temperature and distributes itself in equal shares to each of the independent ways in which the molecules can absorb energy.
(b) By the equipartition theorem the energy will be equally distributed over all molecules. Therefore each mass m_i has a kinetic energy of (1/2)kT and

$$K_{av} = \frac{1}{2} m_i v_{i(rms)}^2.$$

The pressure from each mass species is

$$p_i = \frac{2N_i}{3V} \left(\frac{1}{2} m_i v_{i(rms)}^2\right) = \frac{3}{2} V \sum_{i=1}^{\ell} p_i.$$

But the equation of state is

$$p_{tot} V = (2/3)U.$$

Therefore

$$p_{tot} = \frac{2}{3} \frac{U}{V} = \sum_{i=1}^{\ell} p_i.$$

MASTERY TEST GRADING KEY - Form C

1. Solution: (a) An ideal gas is one that exactly obeys the equation of state $pV = nRT$, where p = pressure, V = volume, n = number of moles, R = gas constant, and T = absolute temperature.
(b) We use the equation of state $pV = nRT$. From the initial conditions we can find n:

$$n = \text{number of moles} = \frac{m}{M} = \frac{5.6 \times 10^{-5} \text{ kg} \times 10^{3} \text{ g/kg}}{28 \text{ g/mole}} = 2.00 \times 10^{-3} \text{ mole.}$$

Therefore

$$p = \frac{nRT}{V} = \frac{(2.00 \times 10^{-3} \text{ mole})(8.31 \text{ J/kg mole K})(300 \text{ K})}{(2.00 \times 10^{-5} \text{ m}^3)} = 2.50 \times 10^{5} \text{ Pa.}$$

2. What To Look For: Ask about one of the answers: for example, random motion - student should tell you it is random motion of the molecules. They move in all directions with equal probability and a variety of speeds.

Solution: (1) Large number of particles; (2) Negligible volume; (3) Random motion; (4) No forces except during collision; (5) Elastic collisions; (6) Newtonian mechanics applies.

3. What To Look For: Look for sign error. Ask student to work out units for final equation. Ask student what θ will give maximum pressure ($\theta = 0$).

Solution: A single molecule hitting the wall changes it momentum by

$$\Delta p\hat{i} = mv_f\hat{i} - mv_i\hat{i} = m(v_f - v_i)\cos\theta = m(-v - v)\cos\theta = -2mv\cos\theta.$$

The change of momentum of the wall is $+2mv\cos\theta$ by conservation of momentum.
The force on the wall is $-\Delta p\hat{i}/\Delta t = 2mv\cos\theta$ (number/s).
The pressure is force per unit area = $2mv\cos\theta$ (number/s $\times$ area) = $2mv\cos\theta$ M.

4. Solution: (a) The kinetic energy per molecule is

$$K_{av} = \frac{1}{2}mv^2_{rms} = \frac{3}{2}kT = \frac{3}{2}kT = (\frac{3}{2})(1.38 \times 10^{-23} \text{ J/mole K})(273 \text{ K}) = 5.6 \times 10^{-21} \text{ J.}$$

(b) The kinetic energy per mole is

$$N_0K_{av} = \frac{1}{2}Nmv^2_{rms} = \frac{3}{2}N_0kT = (6.02 \times 10^{23})(5.6 \times 10^{-21} \text{ J}) = 3.40 \times 10^{3} \text{ J.}$$

5. Solution: (a) The available energy depends only on temperature and distributes itself in equal shares to each of the independent ways in which the molecules can absorb energy.
(b)(i) The specific heat per molecule is $C_v = dU/dt$. For a monatomic gas we can have only three translational degrees of freedom and no rotation or vibrations. Therefore, $U = (3/2)kT$ per molecule, and

$$C(\text{per molecule}) = (d/dt)(3kT/2) = 3k/2.$$

The specific heat per unit mass is

$$C(\text{per mass}) = \frac{C_V(\text{per molecule})}{m(\text{kg/molecule})}.$$

Therefore,

$$m = \frac{(3/2)k}{C_V(\text{per mass})} = \frac{(3/2)(1.38 \times 10^{-23}\ \text{J/K}}{0.314\ \text{J/kg K}} = 6.6 \times 10^{-26}\ \text{kg}.$$

(ii) One atomic weight M is the mass of one mole measured in grams.

Therefore

$$M = mN_0(10^3\ \text{g/kg}) = (6.6 \times 10^{-26}\ \text{kg/atom})(6.02 \times 10^{23}\ \text{atoms/mol})(10^3\ \text{g/kg})$$
$$= 40\ \text{g/mol}.$$

Alternate solution: We can use the specific heat per mole in a parallel solution:

$$C_V(\text{per mole}) = d(U)/dt.$$

From kinetic theory U(per mole) = (3/2)RT. Therefore C_V(per mole) = (3/2)R.

But

$$C_V(\text{per mass}) = \frac{C_V(\text{per mole})}{M(\text{g/mole})}.$$

Therefore

$$M = \frac{3}{2}\left(\frac{8.368\ \text{J/mol K}}{0.314\ \text{J/kg K}}\right) = 40\ \text{g/mol}$$

$$m = M/N_0 = \frac{40}{6.02 \times 10^{23}} = 6.6 \times 10^{-23}\ \text{g} = 6.6 \times 10^{-26}\ \text{kg}.$$